

# **CALCULATION METHODS OF CATHODIC PROTECTION SYSTEM PARAMETERS WITH VERTICAL ANODIC ZINC STRINGS**

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## **ABSTRACT**

*In this paper we analyze protection potential/current distribution for cathodic protection systems (CP systems) with vertical galvanic anode strings. Analytical calculation in case of so called primary current distribution is shown in this paper. In this case, the potential on electrodes, which is the boundary condition, is constant and we only consider the effects of electrical field. Also, the numerical calculation of so called secondary distribution of current and potential is described in the paper. In this case, the boundary conditions on electrodes are nonlinear and alongside the effects of electric field, we must consider the kinetic effects on electrodes. The calculation of current/potential distribution in CP system with galvanic anodes is based on defining the mathematical model of solving Poisson or Laplace equation. Solving of second term partial differential equation in soil and its boundaries requires the precise defining of boundary conditions on anode and cathode surface. Boundary conditions on electrode surfaces represent the functional relationships between current and potential. After the installation of cathodic protection system and multiple year measurement on the given object, the results of numerical calculations of cathodic protection parameters on a concrete object are compared with the measured values. Numerical and measured values correspond to each other.*

## **1. INTRODUCTION**

Modeling of corrosive process implies defining as simple as possible mathematical model of consistency with the basic characteristics of the electrochemical system. If a practical engineering design is concerned, mathematical model can be simplified depending on the required accuracy of the calculation. In the case of complex geometry system, these can be simplified only in cases of objects with a high degree of symmetry, which can be adequately modeled as a one

dimensional or two-dimensional. These simpler models with linear boundary conditions can be subjected to analytical techniques that are well covered in the mathematical theory of fields.

Construction of complex geometry objects that are installed in different corrosion aggressive environments (electrolytes) inevitably results in the introduction of more sophisticated scientific methods of calculation of parameters of cathodic protection system. The development of underground metal infrastructure increases the need for the application of cathodic protection. Intensive application of cathodic protection results in the development of numerical methods for solving distribution of potential /current density.

Defining the spatial distribution of protective current density of cathodic protection is a starting assumption for the optimal design of electrochemical protection of underground metallic structures. Cathodic protection system must be designed so as to ensure uniform distribution of protective current density on the surface of the protected object.

Most often used numerical methods to solve the current density distribution of cathodic protection are: Finite Element Method – **FEM**, Boundary Element Method - **BEM** and coupled method (**BEM/FEM**).

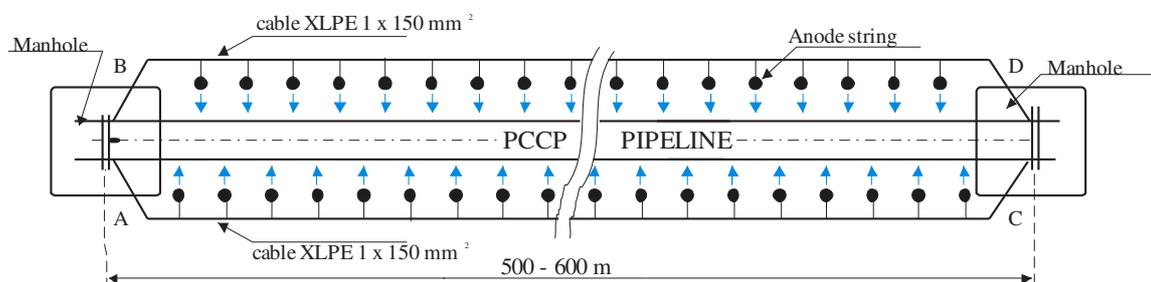
M.E.Orazem [5], C.A.Brebia [7], and D.P.Riemer[6], in his papers described the development of three-dimensional **BEM** numerical calculation methods taking into account the attenuation of the longitudinal potential in the pipeline. R. A. Adey[8], in his papers, implemented a complete **3D** model for the calculation of the potential keeping in mind the effects of interference.

The variety of electrochemical processes in the cathodic protection system is such that there is no general model of the calculation that can cover most of the processes that interest us. In this paper, the numerical calculation was used in a combined **BEM/FEM** method. The method was adapted for the calculation of the distribution of protective potential and current in the extreme conditions of exploitation in terms of solutions (desert sand), taking into account the large-scale system (4000 mm diameter pipe).

## **2. SHORT DESCRIPTION OF CP SYSTEM ON WHICH THE MEASUREMENTS HAVE BEEN MADE**

Cathodic protection system with strings of galvanic anodic electrode on which the calculations and measurements were performed, was applied to water supply system made of reinforced concrete pipes without insulation, 4000 mm in diameter. Concrete covering pipeline with prestressed steel wire is buried at average depth of 7 m. Pipeline (which is the cathode) is placed in the ground (desert sand). Manholes, which represent the ends of section, are accessible for connection of cable installation, control and measuring station and similar. The lengths of sections between manholes is 500-600 m. The complete system is provided with electrical continuity. Metal sheathing for water resistance is set up inside the pipeline. On the outside of the metal sheathing, due to mechanical reasons, prestressed steel wire diameter of 4-6 mm was placed, which is the object of cathodic protection. Cathodic protection system is implemented by setting up in-depth zinc strings alternatively, on both sides of the pipeline, on mutual distance of 6 m.

Fig. 1 shows the principal scheme of cathodic protection system for one pipeline section.



**Fig 1.** CP System for one pipeline section

CP System is realized according to the design solution by placing vertical anodic strings of zinc electrodes at 10 -15 m depth.

Anodic material is zinc alloy with theoretic electrochemical capacity 820 Ah/kg, total mass of 45,14 kg. Dimensions of one anode strings are 6 400 mm ( $\Phi$  150 mm) including backfill. The principle scheme of one anode string with 6 anodes is shown in the Fig. 2.

Anode of zinc alloy (zinc over 99%) shed into molds dimensions of approximately 928 x 35 x 35 mm trapezoid shape, with 4 mm diameter galvanized steel wire passing through its center.

In calculation of the transitive resistance for anode string buried at depth  $h$  is given  $R_{AG} = \frac{\rho}{2\pi l} (\ln \frac{8l}{r} - 1)$ ;  $l \gg d$ . In this paper we used modified functional relationship recommended by manufactures of zinc anodes:

$$R_{AL} = \frac{\rho}{2 \cdot \pi \cdot l} \left( \ln \frac{2 \cdot l}{r_B} + \frac{1}{2} \ln \frac{4h+l}{4h-l} + \frac{\rho_B}{\rho} \ln \frac{r_B}{r_A} \right), \quad (1)$$

where:

$l$  – length of the anode string ( $l=6\,400$  mm),

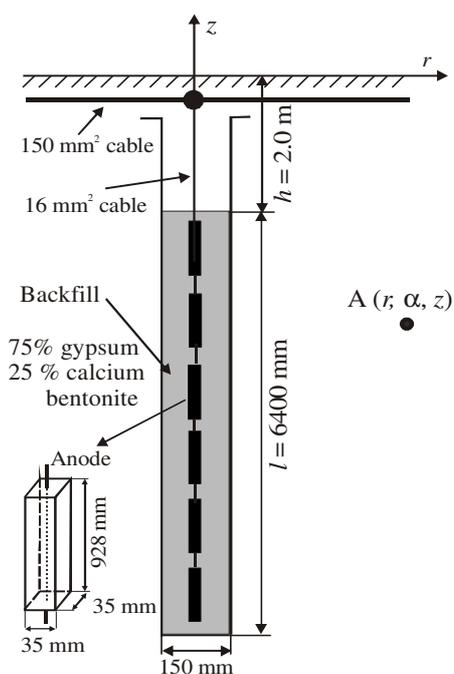
$r_B$  – radius of the anode string with backfill (in calculation  $2r_B = 150$  mm),

$r_A$  – radius of anode string with backfill (in calculation  $2r_A = 35$  mm),

$\rho_B$  – soil resistivity of the backfill (in calculation  $\rho_B = 3,0$   $\Omega$ m),

$\rho$  – soil resistivity of the electrolyte (solution),  $\Omega$ m,

$h$  – average depth of burying anode string (in calculation  $h = 2000$  mm).



**Fig. 2.** Anode string

### 3. CALCULATION OF THE CURRENT DENSITY DISTRIBUTION IN CATHODIC PROTECTION SYSTEM

In the primary distribution of protective current density, only the effects of electric fields are taken into account, and in addition to the secondary distribution of electric field effects kinetic effects on the electrodes are taken into account. Tertiary distribution of current density, in addition to these effects, the concentration gradient is taken into account.

#### 3.1 Calculation of primary current density distribution

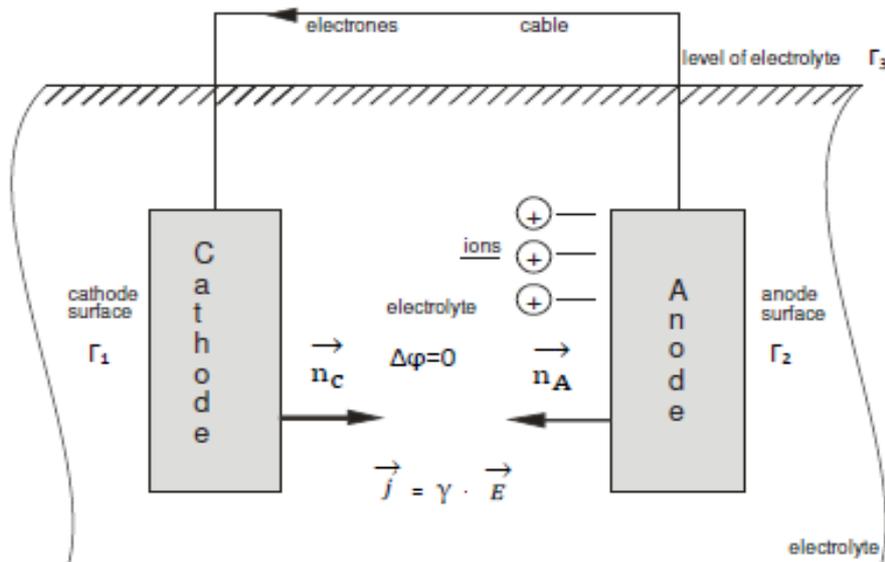
Laplace equation for the solution area can be analytically solved only in the case when it comes to simple geometries of structures with linear boundary conditions at the electrode surface. When solving the given equation, the simplest case of boundary conditions on the electrode surface is to take a constant potential on the electrodes. In this case, Laplace's equation is solved under the following conditions:

$$\varphi = \text{const} \quad (\text{on the surface of electrodes}) \quad (2)$$

$$\frac{d\varphi}{dn} = \text{const} \quad (\text{in solution}) \quad (3)$$

where:  $\vec{n}$  – vector of perpendicular at the boundary area of electrode/solution.

The entire area can be seen as an electrochemical system, as shown in Fig. 3



**Fig. 3.** Schematic representation of the boundary of the integration

Constant potential as a boundary condition is set when the electrodes are practically unpolarized, i.e. when it is:

$$\frac{d\varphi}{d\vec{j}} \rightarrow 0, \quad (4)$$

where:  $\varphi = \varphi_M - \varphi_{el}$  – potential on the electric double layer, V,

$\varphi_M$  – the potential of the protected object - the cathode after polarization, mV,

$\varphi_{el}$  – potential in solution, mV,

$\vec{j}$  – current density vector, A/m<sup>2</sup>.

Reactions on the electrodes take place very quickly so that polarization is only caused by ohmic potential drop in the solution and there is no change in potential electrodes. The distribution of potential and current density depends only on the geometric relationships in the electrochemical system. When the primary current density distribution of the equality is concerned, the equation  $\varphi = \text{const.}$  means that the potential in the electrolyte  $\varphi_{el}$  along the metal surface is solely function of the geometry of the electrochemical corrosion system. In this case, only potential in the electrolyte as a function of the geometric relationships in the electrochemical system is accounted. The only physical quantity which is important for the distribution of current density in the system is the specific electrical conductivity of the electrolyte  $\gamma$ .

This is only possible in cases, if a large enough and long parallel metal surfaces are in concerned, or the concentric equipotential cylindrical electrodes of great length, or the concentric spherical electrodes.

### 3.1.1 Mathematical model of galvanic element for primary current distribution

In the case of a stationary electric field, vector of the current density in linear environments must satisfy Ohm's law:

$$\vec{J} = \gamma \cdot \vec{E} = -\gamma \cdot \text{grad}\varphi \quad (5)$$

and Kirchhoff's first law

$$\text{div}\vec{J} = 0 \quad (6)$$

Inserting the equation (5) in the equation (6), the first Kirchhoff's law can be written as:

$$\text{div}(-\gamma \cdot \text{grad}\varphi) = 0 \quad (7)$$

In the case where the specific electrical conductivity  $\gamma$  constant can be drawn before the operation of divergence, so the relation is true

$$\operatorname{div}(\operatorname{grad}\varphi) = \Delta\varphi = 0 \quad (8)$$

in other words

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 \quad (9)$$

So, in the case of stationary electric field in a homogeneous linear region ( $\gamma$  is independent of the amount and direction of electric field  $\vec{E}$  strength vector) potential function satisfies the Laplace equation.

If there is a strings of anode electrodes shown in Fig. 2. length  $l$ , radius  $r_A$ , which is buried to a depth  $h$  from which the total current intensity  $I$  is flowing out, the distribution of potential can be most easily determined by the change of method of the characters.

By analogy with the electrostatic field potential value at point A ( $r, \alpha, z$ ) (Fig. 2.) is:

$$\varphi(r, \alpha, z) = \frac{I}{4\pi l \gamma} \cdot \ln \left( \frac{\left( z+l+h + \sqrt{(z+l+h)^2 + r^2} \right) \cdot \left( z-h + \sqrt{(z-h)^2 + r^2} \right)}{\left( z-l-h + \sqrt{(z-l-h)^2 + r^2} \right) \cdot \left( z+h + \sqrt{(z+h)^2 + r^2} \right)} \right) \quad (10)$$

It can be seen that the distribution of potential is independent of the angle  $\alpha$ .

The simplified engineering design quite often neglects burial depth  $h$  ( $h \approx 0$ ) so that previous expression can be simplified and we get:

$$\varphi(r, \alpha, z) = \frac{I}{4\pi l \gamma} \cdot \ln \left( \frac{z+l + \sqrt{(z+l)^2 + r^2}}{z-l + \sqrt{(z-l)^2 + r^2}} \right) \quad (11)$$

Quite often, when anode strings is concerned, its resistance of distribution is interesting, which can be obtained on the basis of Ohm law over its potential and strength of the current that flows from it.

Inserting the previous expression  $z = 0$  and  $r = r_A$  the potential of anode strings obtained as:

$$\varphi_0 = \frac{I}{2\pi d \gamma} \cdot \ln \left( \frac{\sqrt{l^2 + r_A^2} + l}{r_A} \right) \approx \frac{I}{2\pi d \gamma} \cdot \ln \left( \frac{2l}{r_A} \right) \quad (12)$$

Now the transitive resistance of anodic groundbed can be obtained as:

$$R_{AG} = \frac{\varphi_0}{I} = \frac{1}{2\pi l \gamma} \cdot \ln\left(\frac{2l}{r_A}\right) \quad (13)$$

Boundary condition on the surface of the pipeline (equipotential surface) can be obtained from the equation:

$$\bar{j}_n = -\gamma \frac{\partial \varphi}{\partial n} = -\gamma \frac{\partial \varphi}{\partial r} = const \quad (14)$$

Primary distribution of surface density of the protective power, when the geometry of the electrochemical system and the operating voltage  $U_P$  galvanic elements are known, is defined as:

$$\bar{j}(r, \alpha, z) = \frac{U_P}{f(r, \alpha, z)} \cdot \gamma = \frac{\varphi_M - \varphi_{cor}}{f(r, \alpha, z)} \cdot \gamma, \quad (15)$$

where:  $\varphi_M$  – the potential of the protected object - the cathode after polarization, mV,

$\varphi_{cor}$  – the corrosion potential of the protected object, mV,

$\gamma$  – specific electrical conductivity of solution, S/m,

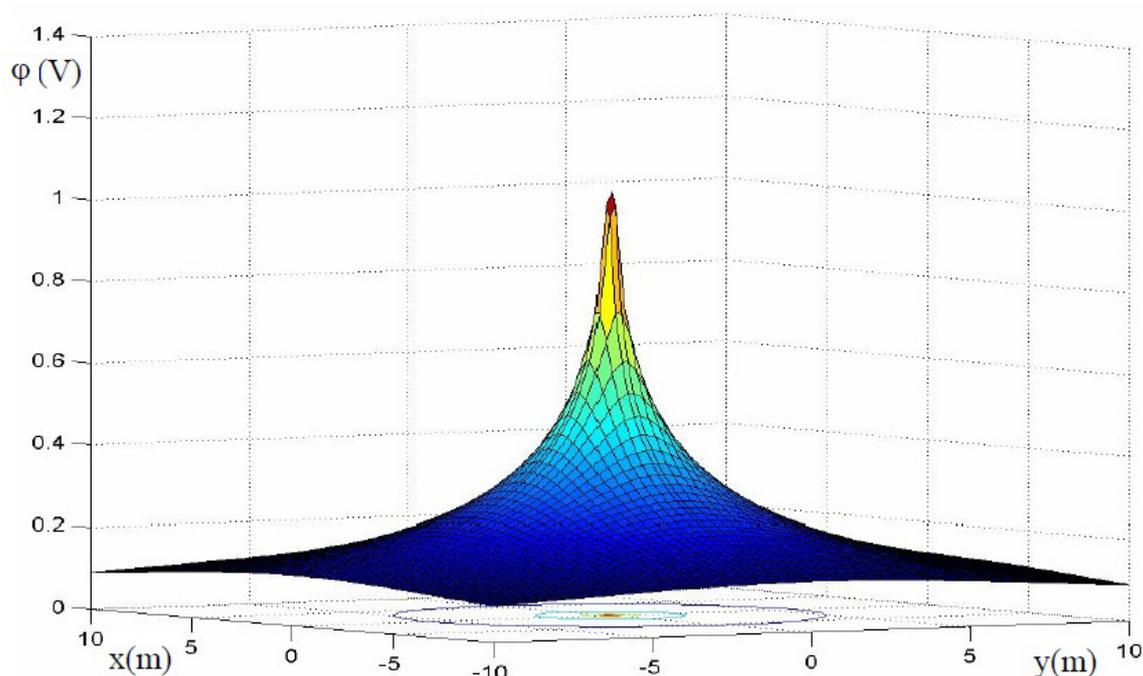
$f(r, \alpha, z)$  – properly chosen function.

### 3.1.2 Results of calculations and measurements

In case the object of specific dimensions, the calculation results are given in the diagram below.

The initial data for a particular case of the calculation are:

- solution is a homogeneous electrolyte with soil resistivity 550,  $\Omega\text{m}$ ;
- total value of the current power provided by the anodic range is 15, mA;
- the length of the anode chain (strings) is 6 400 mm.

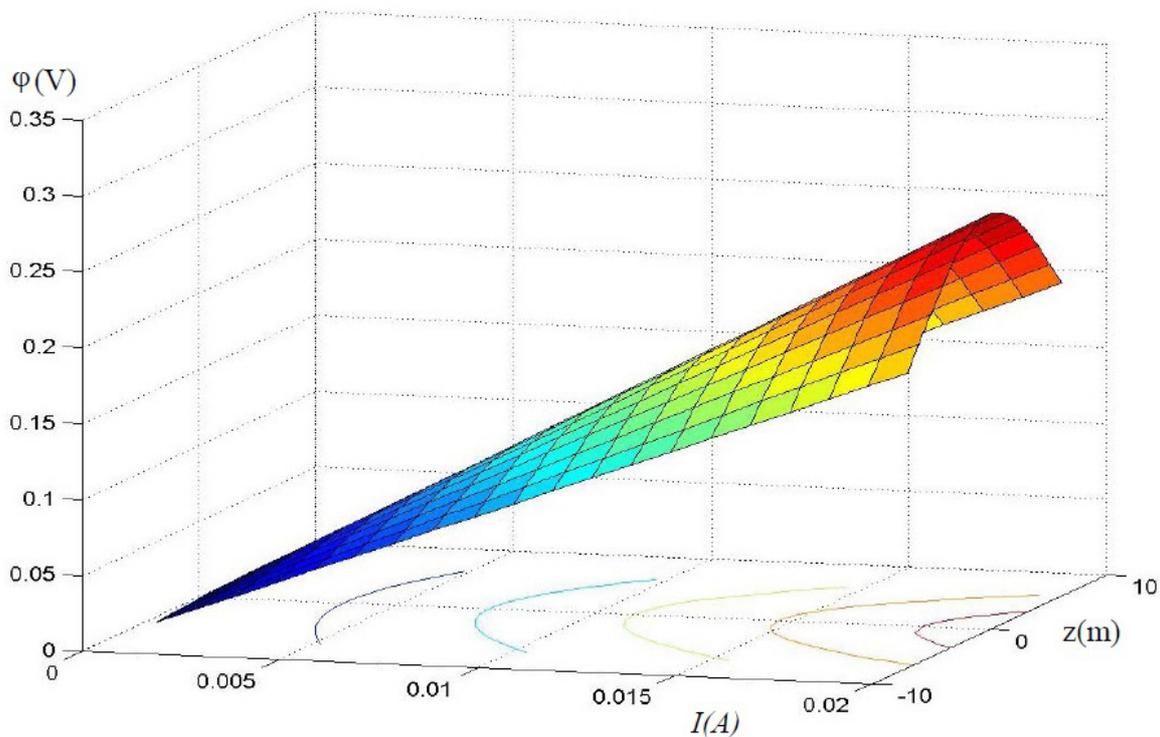


**Fig. 4.** Diagram of the distribution potential of anode string

The diagram in Fig. 4 indicates the distribution of potential on the earth's surface where the coordinate beginning of rectangular coordinate system concurs with the projection of the anode strings on the surface of earth. It shows the distribution of potential on the surface of the earth ( $\varphi_{el}$ ) at the change of  $x$  and  $y$  coordinates of  $0 \text{ (m)} \pm 10 \text{ m}$ . Equipotential lines are concentric circles centered at the coordinates origin. It can be concluded that the potential much faster declines in the vicinity of anode chain than in the case of greater distances from the point of origin.

The given calculation is analyzed in two-dimensional space ( $x, y$ ) for different values of the parameters and on this basis we can conclude the following:

- if the reference electrode (CSE) is placed on the surface of the earth, which is practically the simplest solution, the real value of resources is obtained, for instance of steel pipelines, because there is a voltage drop both in the transition electrode / earth and the voltage drop between ground and pipelines. This is particularly expressed in the large diameter pipelines because it concerns with intervals even greater than 6 m, and as can be seen from the diagram the potential drop is noticeable;
- in case of an electrolyte with high value of soil resistivity the potential decreases more slowly;
- if it concerns with the spatial distribution of protective current density, that is potential, it can be said that it is practically an equipotential surface of cylindrical shape. Then you can ignore the error that occurs because the reference electrode is placed on the earth surface (above the pipeline) because the potential in electrolyte is changed only in the direction that is normal to the axis of anode strings.



**Fig. 5.** Potential changes in the function of current strength change

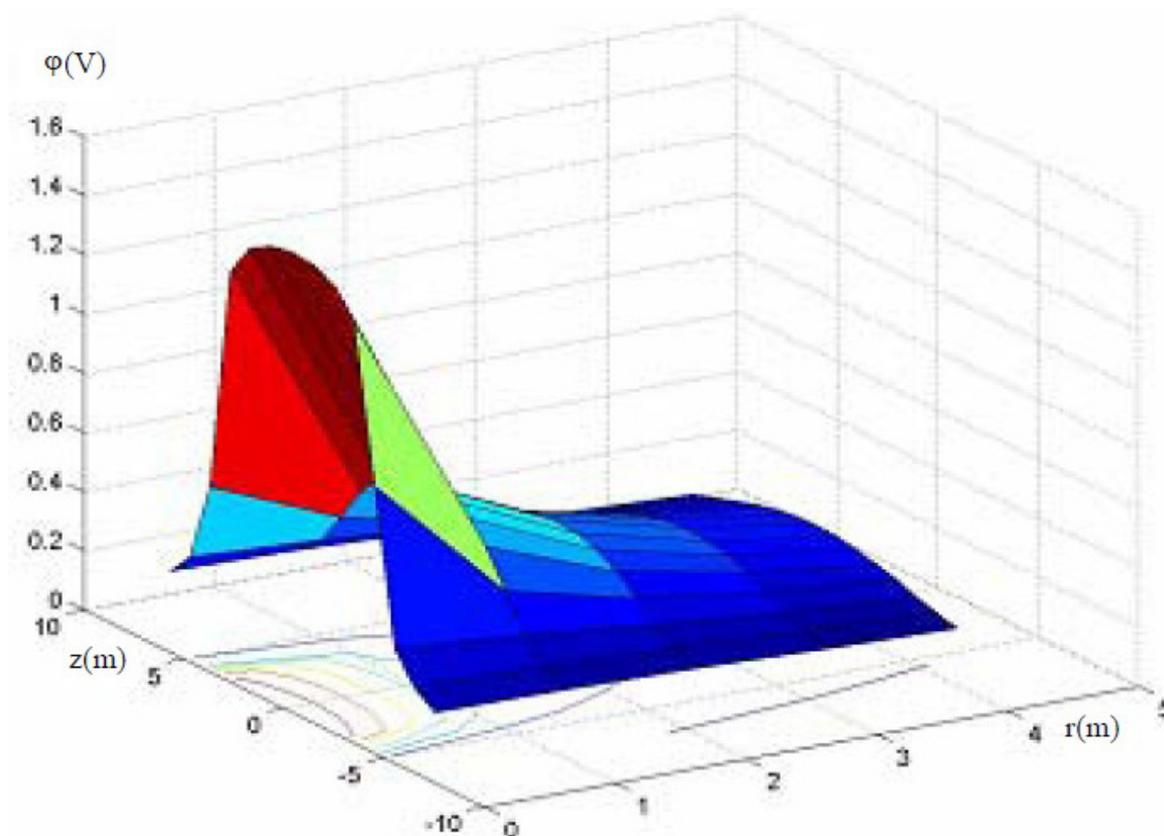
Fig. 5. indicates the potential changes in the electrolyte ( $\varphi_{el}$ ) in depth as a function of changes in the galvanic current anode. It can be seen that there is a linear functional dependence  $\varphi_{el} = f(I)$ . In the middle of a strings of anode a maximum potential value is obtained in the electrolyte.

Fig. 6. indicates the spatial distribution of potential in the electrolyte as a function of the length of anode strings and spacing from the axis of anode chain (string). Analyzing this diagram we can reach the confirmation that it concerns the spatial distribution of resources in the form of an ellipsoid. Equipotential lines are the densest at the ends of anode strings. From this we can derive the following conclusions:

- if the middle of anode string is at a depth of eg. 10 m in the solution, then at that depth there is the least potential drop between the anode and the pipeline which means that it will be the greatest potential on the part of the pipeline which is located at this depth, more precisely on the nearer side of the pipeline, towards the anode;
- as far as the farther side of the pipeline is concerned (provided there are no strings on that side of the anode), for practically it is a non-insulated pipeline (there is no problem with the potential loss in isolation, and thus with the polarization) only the increase in space  $r$  is taken into account.
- for the above reasons it is important to make proper arrangement of anodes on both sides of the pipeline.

If we examine the potential value at different depths, as for instance, at the distance of 6 m from the anode strings, it can be seen that this potential has a maximum value at a depth that

corresponds to the depth of the middle of setting up a strings of anode. Such a distribution is the result of field distribution, that is, the structure of the established lines. In the event that the pipeline is at that distance, which is intended to be polarized, in addition to this potential defined by equation (10) it is necessary to take into account the potential, which is the result of existence of the polarization resistance which takes into account the value of polarization resistance of an object that is protected with cathodic protection.



**Fig. 6.** Distribution of potential in the function of anode strings length

### 3.2 Calculation of secondary current density distribution

When the speed of the reaction on the electrodes is not so large, so that we cannot ignore the polarization, it is necessary to use the secondary protective current density distribution. When calculating the secondary distribution of current density it is also necessary to include into calculation the kinetics of electrode reactions. Influence of electrode kinetics on the distribution of current density and potential is introduced via boundary conditions at the anode and cathode, which are given by Butler - Volmer equations applicable to each specific case.

Thus, for the model of the secondary distribution of protective current density, Laplace's equation determines the potential in the electrolyte (solution), but the boundary conditions at the electrode surface must be taken into account, because it is not about constant potential value. The analytical expressions that define the boundary conditions in the case of the

cathodic protection system, generally, are not linear because of electrochemical reactions on the electrodes do not give a linear functional relationships between the protective current density and corresponding potential:

$$\bar{j}_{nA} = -\gamma \cdot \frac{\partial \varphi}{\partial n} = f_A(\varphi) ; \quad \bar{j}_{nC} = -\gamma \cdot \frac{\partial \varphi}{\partial n} = f_C(\varphi) \quad (16)$$

In this case it is the polarization of the cathode surface with a far more balanced distribution of current density on the surface than when it treats only the primary distribution, namely for the same geometric configuration of the system. In some middle speed response, current distribution and potential depends on the absolute size of the electrodes as well as the geometric ratio of linear dimensions of electrodes.

In ideal polarized electrodes it is  $\frac{d\varphi}{dj} \rightarrow \infty$ .

Speed of the reaction is practically zero, the density of the protective current is evenly distributed in its entirety and does not depend on the geometry of the system if there is no interference from other processes.

### 3.2.1 Nonlinear boundary conditions on the electrode surfaces

Partial electrochemical reactions that take place simultaneously at the cathode surface, which are interesting for corrosion of steel are given in the following equations

1.  $Fe \rightarrow Fe^{2+} + 2e^-$  partial anode reaction at the cathode
2.  $O_2 + 2H_2O + 4e^- \rightarrow 4OH^-$  reaction of oxygen reduction
3.  $2H_2O + 2e^- \rightarrow H_2 + 2OH^-$  partial cathode reaction at the cathode

These reactions take place simultaneously on the surface of steel in real corrosion conditions. The total value of current polarization on the cathode surface is defined by the equation:

$$j_{totalC} = j_{0Fe} \cdot 10^{\frac{\varphi - \varphi_{Fe}}{\beta_{Fe}}} - j_{lim,O_2} \cdot \left( 1 + 10^{\frac{\varphi - \varphi_{O_2}}{\beta_{O_2}}} \right)^{-1} - j_{0H_2} \cdot 10^{\frac{-(\varphi - \varphi_{H_2})}{\beta_{H_2}}}, \quad (17)$$

where:

$j_{0Fe}$  - current density corresponding to the reaction of metal solution  $Fe \rightarrow Fe^{+2} + 2e^-$ ,

$j_{0H_2}$  - current density corresponding to hydrogen evolution reaction  $2H_2O + 2e^- \rightarrow H_2 + 2OH^-$ ,

$j_{lim,O_2}$  - current density corresponding to oxygen reduction reaction  $O_2 + 2H_2O + 4e^- \rightarrow 4OH^-$ ,

$\varphi = \varphi_m - \varphi_{el}$  - potential of the metals vs. CSE (Copper Sulphate Electrode),

$\varphi_m$  – potential of the protected object,

$\varphi_{el}$  – the potential of the soil adjacent to the metal vs. CSE,

$\beta_{Fe}$ ,  $\beta_{O_2}$  i  $\beta_{H_2}$  – Tafel's coefficients (slopes),

$\varphi_{Fe}$ ,  $\varphi_{O_2}$  i  $\varphi_{H_2}$  – free corrosion potential for the corresponding reactions, respectively.

An electrochemical reaction at the anode surface is:



This reaction corresponds to an analytical expression for the polarization current:

$$j_A = j_{0O_2} \cdot \left( 10^{\frac{\varphi - \varphi_{corA}}{\beta_A}} - 1 \right), \quad (19)$$

where:  $j_{0O_2}$  – mass transfer limited current density for oxygen reduction (A/m<sup>2</sup>),

$\beta_A$  – the Tafel's coefficient (slope) (V/dec),

$\varphi_{corA}$  – corrosive potential for reaction at anode (V).

Based on the previous equations we can conclude that it concerns to highly nonlinear polarization diagrams, especially at the cathodic surface.

### 3.2.2 Boundary Conditions with Nonlinear Polarization Characteristics

If we analyze the polarization characteristics of the cathodic steel surface in the cathodic protection system applied to the galvanic anode zinc, it can be seen that the diagram for the total current is nonlinear. This nonlinearity is even greater the less is deviation (overvoltage or polarization) of the corrosion potential. In real conditions it is the corrosive potential of approximately -0.2 V vs. CSE. Combining zinc anode to cathode surface, the same is polarized about 200-300 mV, which means that a new potential for cathodic area of approximately -0.4 to -0.5 V vs. CSE is established. In the range from -0.2 V to -0.5 V polarization characteristic is completely nonlinear. Nonlinearity has a greater significance for practical calculation in the case of larger Tafel slopes and the lower insulation of steel.

In the case of large nonlinearities it must be taken into account the lower and upper limits of the potential which the calculation of current is carried out and this is most often solved with the introduction of the current axis logarithmic.

Fig. 8. clearly explains the physical model of analysis of the electrochemical electrode reactions and their effects on the polarization diagrams of anodic and cathodic areas

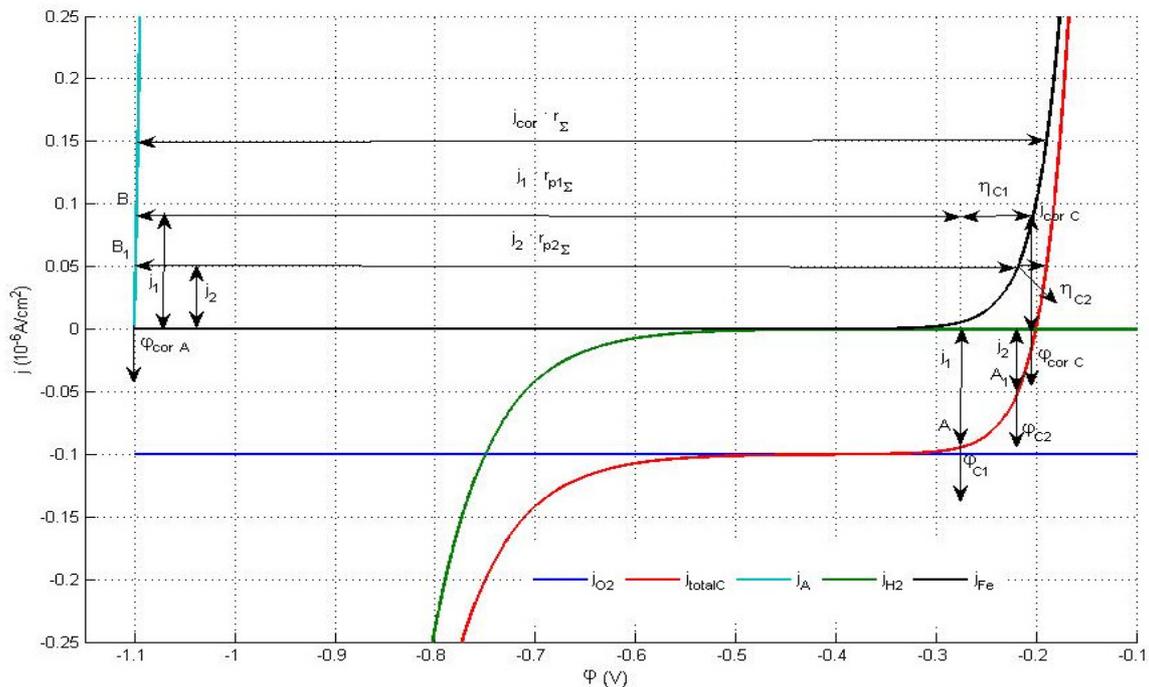
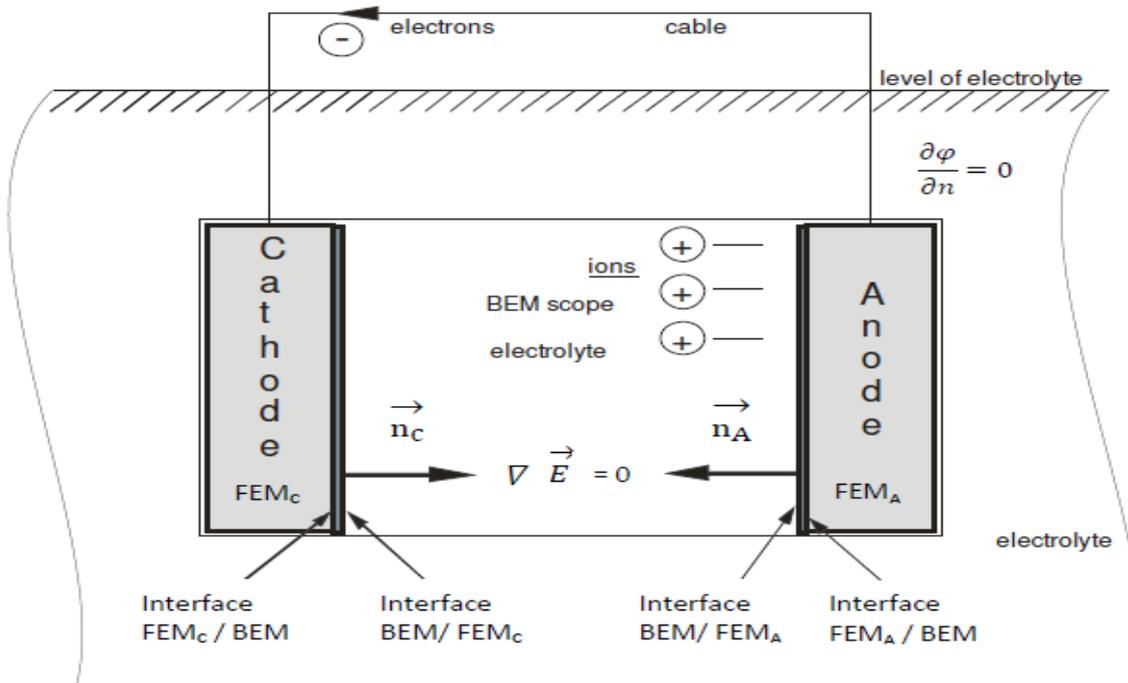


Fig. 7. Polarization chart of galvanic cell/element steel-zinc

### 3.2.3 Mathematical Model of galvanic element for secondary current distribution

For the calculation of the potential distribution in CP system the Finite Element Method (**FEM**), the Boundary Elements Method (**BEM**) and the coupled method (**BEM/FEM**) have been used. Fig. 8. shows the principle of the CP system. The used methods in some areas have been noted in Fig. 8.

The Direct Boundary Elements Method is appropriate for solving stationary fields of electrical current in electrolyte (solution) with infinite boundaries, therefore this method has been used for calculation of protection potential in electrolyte. On the other hand, the FEM method is appropriate for solving the field within limited areas and therefore this method has been used for calculations within the metal surfaces (electrodes). Potential distribution at the electrode/electrolyte boundary (interface) was calculated using the coupled **BEM/FEM** method. It was calculated iteratively using the so-called advanced sequential Dirichlet-Neumann procedure of successive under relaxations.



**Fig. 8.** Principle chart of cathodic protection system

### 3.2.3.1 The Finite Element Method

In the Finite Element Method the observed domain of the physical system (in this case, the electrodes) is divided into a finite number of elements of a certain geometry. The Solving of the field equations is made for each finite element. Laplace's Partial Differential Equation (LPDE) of stationary current field in observed domain is:

$$\frac{\partial}{\partial x} \left( \gamma \frac{\partial \varphi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \gamma \frac{\partial \varphi}{\partial y} \right) + \frac{\partial}{\partial z} \left( \gamma \frac{\partial \varphi}{\partial z} \right) = 0, \quad (20)$$

where:  $\varphi$  - unknown function of potential distribution, V,

$\gamma$  - specific electric conductivity of solution, S/m.

After Galerkin's weighted residuals procedure for the calculation of potential distribution is applied, the following system of linear algebraic equation can be written in the matrix form:

$$[H]^{FEM} \cdot \{\varphi\}^{FEM} = \{Q\}^{FEM}, \quad (21)$$

where:

$[H]^{FEM}$  - two-dimensional matrix of coefficients written for one 3 - D element where the common term is given by:

$$h_{ij}^{FEM} = \sum_{e=1}^{n_e} \left[ \gamma \int_{V^e} \left( \frac{\partial N_i^e}{\partial x} \frac{\partial N_j^e}{\partial x} + \frac{\partial N_i^e}{\partial y} \frac{\partial N_j^e}{\partial y} + \frac{\partial N_i^e}{\partial z} \frac{\partial N_j^e}{\partial z} \right) dV \right] \quad (i = 1, 2, \dots, n_f; j = 1, 2, \dots, n_f), \quad (22)$$

$V^e$  – volume of  $e$ th finite element,

$n_e$  – total number of finite elements,

$n_f$  – total number of finite element nodes in which interpolation of potential is made,

$\{\varphi\}^{FEM}$  – matrix vector column of unknown electric potentials,

$\{Q\}^{FEM}$  – matrix vector column of the free terms where the common term is given by:

$$q_i^{FEM} = - \sum_{e=1}^{n_e} \left[ \sum_{j=1}^{n_f} \left( \int_{S_{\Delta}^e} \gamma N_i^e \cdot N_j^e \cdot \frac{\partial \varphi_j^{FEM}}{\partial n} dS \right) \right], \quad (23)$$

$S_{\Delta}^e$  – boundary surface of  $e$ th element,

$\frac{\partial \varphi_j^{FEM}}{\partial n}$  – Neumann boundary condition,

$N_j^e(x, y, z,)$  – shape functions that are used for interpolation of potential on the  $e$ th element in the following way:

$$\varphi = \sum_{j=1}^{n_f} N_j^e \cdot \varphi_j^e, \quad (24)$$

$\varphi_j^e$  – value of potential in the nodes of the finite element.

Solving the Galerkin's weighted residuals system of equations, taking into consideration the boundary conditions, gives the values of potential in the nodes of the finite element mesh. The coefficient matrix  $\mathbf{H}$  is a seldom filled matrix and therefore, in order to solve the matrix system of

equations, the use of Crout's or Dolittle's factorization method and the forward-backward switch

algorithm together with the technique of rarely filled matrices is useful.

### 3.2.3.2 The Direct Boundary Element Method

The Boundary Element Method, also known as method of moment, is appropriate for solving the field in infinite or semi-infinite domains. Mathematical model of the direct **BEM**

method is based on Green's symmetrical identity as well as equations of continuity. These equations serve to apply boundary conditions on the boundaries between different media. Let us consider two different cases of calculating **3D** stationary currents fields:

- the case when the observed point **Q** falls into domain **V**,
- the case when the observed point **Q** falls on boundary media.

Volume **V**, which is bounded with surface **S** is observed. Field source with potential  $\varphi(\mathbf{P})$  is at the point **P**, and at the point **Q** the function of potential  $\varphi(\mathbf{Q})$  is observed.

The general formula for potential calculation inside, on the boundary, and outside of the domain is given with:

$$C(\mathbf{Q}) \cdot \varphi(\mathbf{Q}) + \int_S T(\mathbf{P}, \mathbf{Q}) \cdot \varphi(\mathbf{P}) \cdot dS_p = \int_S G(\mathbf{P}, \mathbf{Q}) \cdot \frac{\partial \varphi(\mathbf{P})}{\partial n_p} \cdot dS_p, \quad (25)$$

where:

$G(\mathbf{P}, \mathbf{Q})$  - the Green function, chosen depending on the type of the coordinate system,

$T(\mathbf{P}, \mathbf{Q})$  - derivative of Green's function in direction of outward normal vector  $\bar{n}_p$  to boundary surface,

$\varphi(\mathbf{Q})$  – function of potential in the observed point **Q**,

$\varphi(\mathbf{P})$  i  $\frac{\partial \varphi(\mathbf{P})}{\partial n}$  – function of potential and its normal derivation in point of source **P**,

$C(\mathbf{Q})$  – constant that can acquire different values:

$$C(\mathbf{Q}) = \begin{cases} 1 & \text{inside domain } V \text{ (Poisson formula)} \\ \frac{1}{2} & \text{on smooth boundary in 3 - D domain} \\ \frac{\alpha_{3-D}}{4\pi} & \text{on discrete boundary of areal angle } \alpha_{3-D} \\ 0 & \text{outside of domain } V \end{cases} \quad (26)$$

$dS_p$  – elementary surface of boundary element of field source discretization  $\varphi(\mathbf{P})$  and  $\frac{\partial \varphi(\mathbf{P})}{\partial n}$ ,

**S** – boundary surface of observed domain.

The integration of weighted residuals function can be avoided by using the collocation point procedure. Dirac's delta function in this procedure is the weighted function. Collocation points are at nodes  $Q_i$  of boundary elements (i.e. at points where potential  $\varphi(\mathbf{Q})$  is observed). But a system of  $n$  equations for  $n$  nodes of boundary elements must be written.

After the appropriate mathematical procedure is applied, we get the following system of algebraic equations:

$$[H]^{BEM} \cdot \{\varphi\}^{BEM} = [G]^{BEM} \cdot \left\{ \frac{\partial \varphi}{\partial n} \right\}^{BEM}, \quad (27)$$

where:

$[H]^{BEM}$  - two-dimensional of matrix, which are composed from two parts, where general term is define by:

$$h_{i,j}^{BEM} = \sum_{e=1}^{n_e} \int_S N_j^e \cdot T_{i,j}^e \cdot dS_p + \delta_{i,j} \cdot C_i \quad (i = 1,2,\dots,n_j ; j = 1,2,\dots,n_e) \quad (28)$$

$[G]^{BEM}$  - two-dimensional matrix of which general coefficient is defined as follows:

$$g_{i,j}^{BEM} = \sum_{e=1}^{n_e} \int_S N_j^e \cdot G_{i,j}^e \cdot dS \quad (i = 1,2,\dots,n_j ; j = 1,2,\dots,n_e) \quad (29)$$

$\{\varphi\}^{BEM}_i \left\{ \frac{\partial \varphi}{\partial n} \right\}^{BEM}$  – matrix vector column of variables,

$$\delta_{i,j} - \text{Dirac's delta function defined as } \int_V \delta(r) dV = \begin{cases} 1 & \text{za } r \in V \\ 0 & \text{za } r \notin V \end{cases} \quad (30)$$

$i$  – index related to the observing point Q,

$j$  – index related to the source point P.

### 3.2.3.3 Coupled method BEM/FEM

On solution/air or solution/dielectric (when it is a boundary of solution and another solution with high resistivity) boundary, so called boundary of the **BEM** domain, only one variable value (potential  $\varphi$  or its derivate  $\partial/\partial n$ ) is known. For example, on the solution/air (earth surface) boundary the homogenous Neumann's condition is given  $\partial\varphi/\partial n_p = 0$ , so  $\varphi$  is calculated.

In this case an equation system is made for each boundary of the domain, taking into consideration Dirichlet's and Neumann's boundary conditions. On the **FEM/BEM** boundary, additional equations of continuity for  $\varphi$  or  $\partial/\partial n_p$  are written.

In this paper, the advanced sequential Dirichlet-Neumann **BEM/FEM** algorithm is used. This algorithm consists of the following steps:

- partitioning of the whole domain into **BEM** and **FEM** domains,
- defining initial potential values on the **BEM/FEM** boundary,
- starting the iterative procedure which lasts until the set convergence condition is fulfilled.

### 3.2.3.4 Calculation of electric field in BEM domain

Dirichlet or Neumann boundary conditions are set for boundary of the **BEM** domain, except on the boundary between the **BEM** and **FEM** domains. In this case the matrix system (21), considering variables  $\varphi$  or  $\partial/\partial n_p$  on both boundary sides (**BEM**)/( **BEM/FEM**), could be written in the following form:

$$[H]^{BEM} \begin{Bmatrix} \{\varphi\}_{n+1}^{BEM} \\ \{\varphi\}_{n+1}^{BEM / FEM} \end{Bmatrix} - [G]^{BEM} \begin{Bmatrix} \left\{ \frac{\partial \varphi}{\partial n} \right\}_{n+1}^{BEM} \\ \left\{ \frac{\partial \varphi}{\partial n} \right\}_{n+1}^{BEM / FEM} \end{Bmatrix} = \{0\} \quad (31)$$

Where:  $\{0\}$ – column null-matrix.

In the system (27) it is necessary to take into account the Dirichlet and Neumann boundary conditions including the potential values at the **BEM/FEM** boundary from the previous iteration step. By calculating the system (27), the normal components of the electric field at the **BEM/FEM** boundary will be found:

$$\left\{ \frac{\partial \varphi}{\partial n} \right\}_{n+1}^{BEM / FEM} \quad (32)$$

### 3.2.3.5 Calculation of continuity equation at BEM/FEM boundary

On the interface between the finite and boundary elements  $S_{BEM} - S_{FEM}$ , the equation of continuity must be satisfied:

$$\gamma^{FEM} \left\{ \frac{\partial \varphi}{\partial n} \right\}_{n+1}^{FEM / BEM} = -\gamma^{BEM} \left\{ \frac{\partial \varphi}{\partial n} \right\}_{n+1}^{BEM / FEM} \Rightarrow$$

$$\left\{ \frac{\partial \varphi}{\partial n} \right\}_{n+1}^{FEM / BEM} = -\frac{\gamma^{BEM}}{\gamma^{FEM}} \left\{ \frac{\partial \varphi}{\partial n} \right\}_{n+1}^{BEM / FEM}$$
(33)

For cathodic and anodic surfaces polarization curves are taken into account, as follows:

$$j_{totalC} = \gamma \left\{ \frac{\partial \varphi}{\partial n} \right\}_{n+1}^{BEM / FEM} ; \quad j_{totalA} = \gamma \left\{ \frac{\partial \varphi}{\partial n} \right\}_{n+1}^{BEM / FEM}$$
(34)

As result we get the Neumann boundary conditions that are applicable in the direction of the normal vector on the **FEM** side of the **BEM/FEM** interface at the anode and the cathode.

### 3.2.3.6 Calculation of electric field in FEM domain

In this step the electric field in the **FEM** domain is calculated. The Dirichle or Neumann boundary conditions are set for the **FEM** domain boundary except for the **FEM/BEM** boundary itself. Therefore, considering the variables  $\varphi$  or  $\partial\varphi/\partial n$  on both boundary sides (**FEM**)/(**FEM/BEM**), the matrix system (21) can be written in the following form:

$$[H]^{FEM} \cdot \begin{Bmatrix} \{\varphi\}_{n+1}^{FEM} \\ \{\varphi\}_{n+1}^{FEM / BEM} \end{Bmatrix} = \begin{Bmatrix} \{Q\}_{n+1}^{FEM} \\ \{Q\}_{n+1}^{FEM / BEM} \end{Bmatrix},$$
(35)

In which elements of matrix  $\{Q\}_{n+1}^{FEM}$  are calculated applying the following expression:

$$q_i^{FEM} = - \sum_{e=1}^{n_e} \left[ \sum_{j=1}^{n_f} \left( \int_{S_d^c} \gamma \cdot N_i^e \cdot N_j^e \cdot \frac{\partial \varphi_j^{FEM}}{\partial n} \cdot dS \right) \right]$$
(36)

As result, we get potential  $\{\phi\}_n^{FEM / BEM}$  at the boundary **FEM/BEM**.

### 3.2.3.7 Correction of calculated potentials at BEM/FEM boundary

In this step, the correction of the potentials calculated in the previous step at the **FEM/BEM** boundary is performed. The correction is made by using the successive under-relaxation method:

$$\{\varphi\}_{n+1}^{BEM/FEM} = (1 - \theta) \cdot \{\varphi\}_n^{BEM/FEM} + \theta \cdot \{\varphi\}_{n+1}^{FEM/BEM} \quad (37)$$

In which  $\theta$  is a factor of sub-relaxation within intervals from 0 to 1.

For numerical calculation, a computer program was made. Descriptive coupled method of final and edge elements was used in such program. Complete calculation was performed in the mentioned program. The mentioned program checks convergence of iterative cycles and it stops when adequate accuracy is reached.

## 3.2.4 Results of numerical calculations and measurements

Numerical calculations applying software were performed for system of cathodic protection with galvanized zinc anodes (such system of protection was applied to the object described in the previous section). This study gives calculation made for one section of approximately 30 m length. It is supposed that electrolyte represents a homogenous soil that has a soil resistivity amounting to 500  $\Omega\text{m}$ . Graphical results of numerical calculation are listed hereafter. During installation of cathodic protection system and after that in the course of its exploration, periodical measurement of potential and current density were made at the same part of the section. Results of measurements of potential are listed hereafter.

### 3.2.4.1 Results of numerical calculations

Fig. 9. illustrates distribution of protective potentials in 3D space at one section of 18 m. Distance between two anode chains at the same side of pipeline is amounting to 12 m. Most negative value of potential in electrolyte is in a close area of anode string.

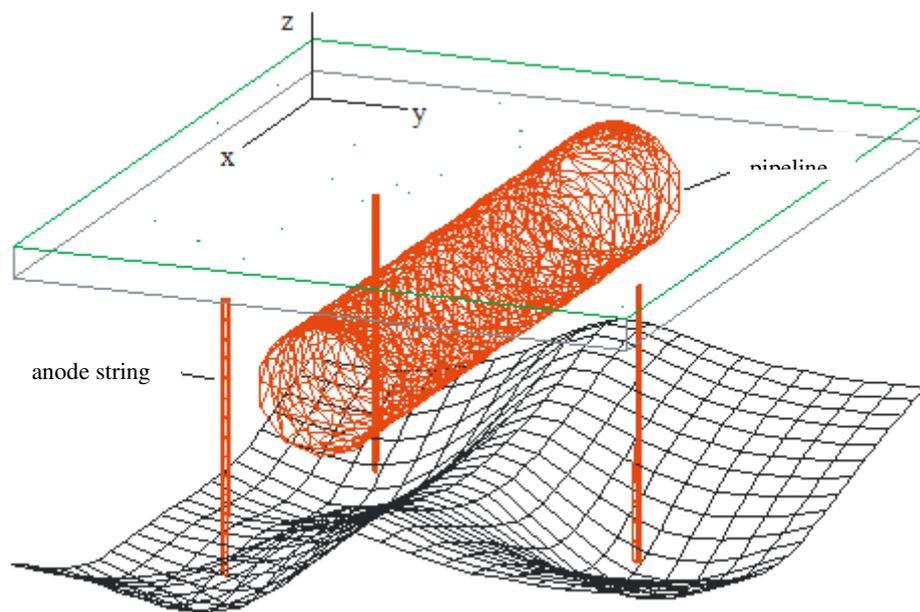


Fig. 9. Distribution of protective potentials

Longitudinal profile of pipeline having adequate calculation of potential distribution is shown by Fig. 10. Such way of giving calculation values of potential distribution is suitable for defining mutual distance of anode chains (strings) (on figure shown as AN) i.e. defining criteria application of minimum protective potentials along the object.

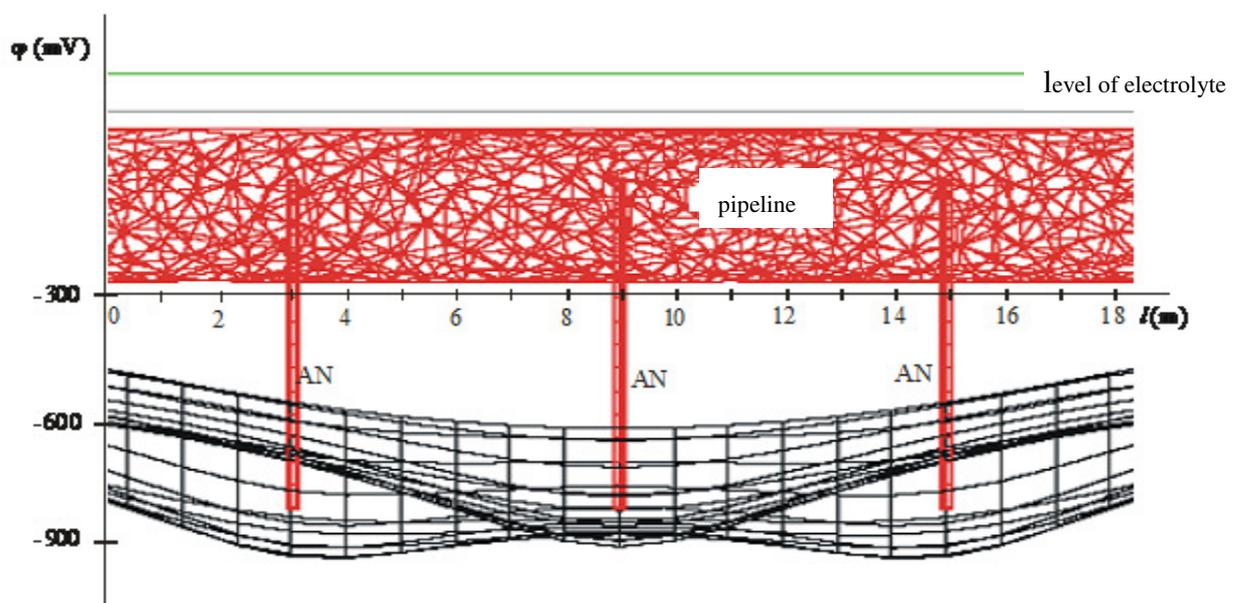


Fig. 10. Distribution of potentials (longitudinal cross-section)

If we compare potential distribution as of the Fig. 10. with measured values of protective potentials as of the Fig. 12., we find out that there is a relative large harmonization of their results. Namely, there are some discrepancies resulted from damage of concrete coating during the stage of pipeline installation, change in value of soil resistivity etc.

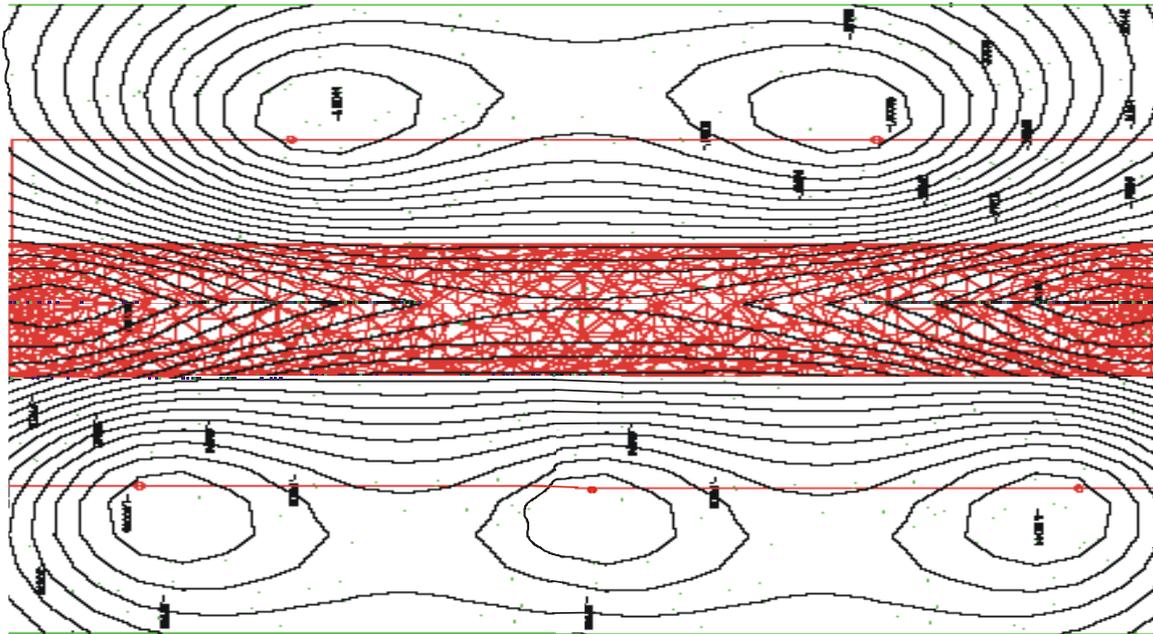


Fig. 11. Potential distribution at the ground surface

The most negative potential value is in the places where anode electrode strings are located, which is logical. On the diagram it can be seen that in those parts the potential reaches values up to -1000 mV at the anode point range. Likewise, we see that the potential being over the pipeline itself that is the closest to the anodic strings equals approximately -600 mV. Potential values above the pipeline change within limits -550 to -600 mV, which is so far quite good agreement with measured values given in Fig. 12.

### 3.2.4.2 Results of measurements

Fig. 12. OFF potential is a measured the potential value of pipeline in relation to the CSE reference electrode immediately after the disconnection of cathodic protection, and ON potential represents the potential value of the pipeline measured when the pipeline cathodic protection system is in operation.

From the Fig. 12. it can be seen that the values, which were calculated by using a numerical program, deviate from the initial ON potential for less than 20%. The numerical calculation of the distribution of potential was made for a homogeneous electrolyte, neglecting the impact of resistance to the connected lines and neglecting the mutual grounding effect of anode strings, etc., so that deviations from the measured values can be considered satisfactory.

Similarly, one can conclude that during the operation of cathodic protection system creating a significant decline of ON potential. ON potential after 18 months of the system

operation have decreased approximately 35% of its initial value. At the moment of putting the system into operation, the protective current density is greater because the operating voltage is also higher. During the time of operation, the operating voltage is decreasing, that is, the potential of protected object becomes less negative. This happens because of increase in polarization resistance of pipeline, increase of resistance of distribution of anode strings, that is, because of a reduction in the total protective current density. Protective current density also decreases due to the existence of voltage drop in cables, anode strings, etc.

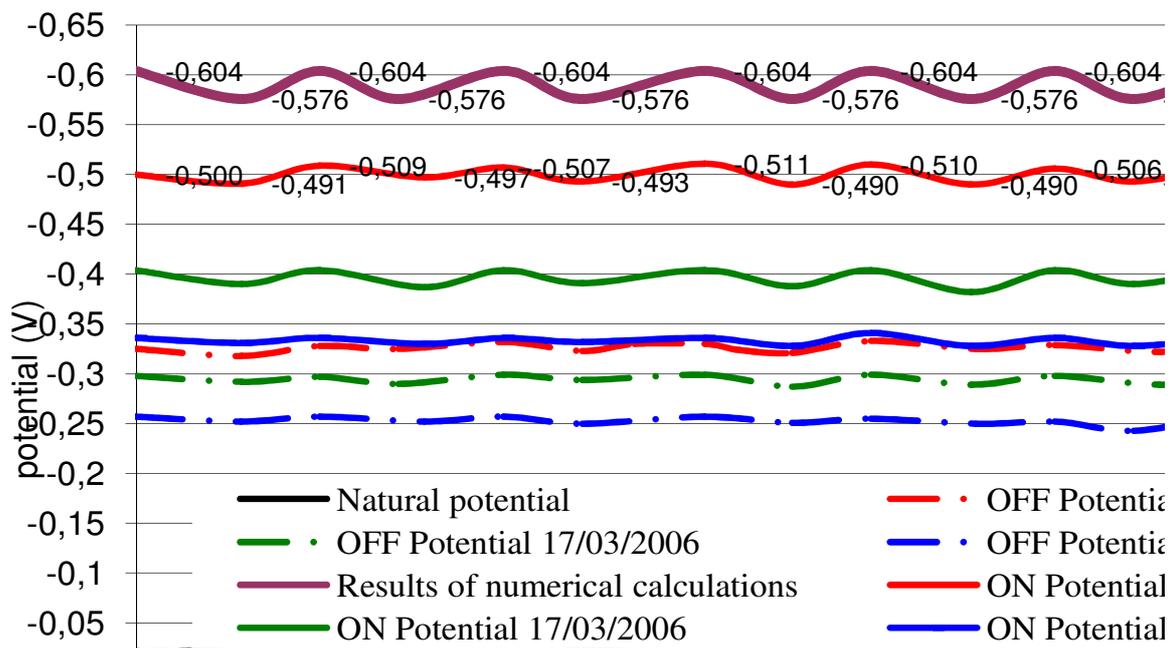


Fig. 12. Results of potential measurement

It may be noted that the difference between ON and OFF potentials will decrease over time. This difference is in fact IR voltage drop in the electrolyte. This is a consequence of the establishment of polarization, that is, the polarization resistance increase, provided we have in mind that the time of polarization of such objects are much longer than in the case of buildings with higher quality insulation.

The diagram shows that even after the exploitation period of 18 months, biasing potential in relation to the natural is higher than 200 mV on the negative side. According to international regulations (eg NACE standard for protection of pipeline is enough to shift the potential, when it eliminates the IR component, in relation to the natural potential to be a minimum of 100 mV after depolarization after 4 hours). Based on this criterion it can be seen that the referred pipeline is cathodically protected in accordance with the regulations.

## 4. CONCLUSION

Used numerical methods for calculating the distribution of protective current / potential in the cathodic protection system with galvanic anodes give the results that do not deviate much from the measured values. Any significant deviations of calculated values from the measured values of the protective potential can be caused by:

- changing in the electrical quality of the concrete lining of pipelines
- changing in the value of the soil resistivity,
- changing the distance between anode strings formed at the stage of installation,
- presence of other underground metallic structures, etc.

Of course, the occurrence of significant deviations of the measured values of the protective potential, requires the calculated potential to be further analyzed so as to identify the causes.

Given the numerical methods are particularly suitable for calculating the protective potential / current of long underground metal objects of large dimensions. It is suitably to show the calculation values as given in Fig. 12. for easier comparison with the measured values.

Solving the distribution of protective potentials / currents in the application of cathodic protection system with a large number of galvanic anodes can be done only by using numerical methods. The paper applied a combined **BEM / FEM** method. The application of this method has great practical importance for designers of cathodic protection system in all conditions, especially bearing in mind the different characteristics of the electrolyte. Then we get correct technical data on the distribution of protective potentials, which practically is the foundation for optimal design of cathodic protection system.

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