MATHEMATICAL MODEL FOR CATHODIC PROTECTION OF THE UNDERGROUND PIPELINES

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ABSTRACT

Cathodic protection is most often used technique for protection of underground or underwater metallic infrastructures from corrosion. Design of the cathodic protection system implies determination of the potential distribution on the protected object surface which must meets given criterions. This paper presents a mathematical model for calculating cathodic protection system parameters. Presented mathematical model was used to conduct the analysis of the impact of soil resistivity and coating resistivity on the level of polarization.

1. INTRODUCTION

Corrosion of metals can be defined as destruction of the metal caused by exposure to and interaction with its environment. Metal can be protected from corrosion by application of various methods. However, technical practice has shown that one of the most effective methods to protect metal structures from corrosion is the application of cathodic protection (CP) system in combination with a passive protection (insulating coatings based on polymers or concrete) [1-6].

Cathodic protection is technique for slowing down or completely stopping of all types of corrosion of underground or underwater metallic structures [7]. Cathodic protection system is assumed to be effective when the rate of corrosion of the protected object does not exceed a defined value, after installation of the CP system. In practice, corrosion protection is achieved by the shifting the potential of underground metal structures to negative side (the polarizing) [1]. Therefore, design of CP system requires defining polarization level of protected object after installation of cathodic protection system. This parameter must meet given criterions during the exploitation of CP system [6].

For calculation of CP system parameters numerous analytical [8,9], semy-analytical [5,6,10,11] and numerical methods have been proposed. Since calculation of the CP system parameters is surface related problem, boundary element method (BEM) is most suitable numerical method for solving this type of problems [12,13]. This method requires only a discretization of boundary surface unlike other numerical methods such as finite difference method (FDM) and finite element method (FEM). Furthermore, there is no need for discretization of the boundaries ground-air as well as the boundaries between two adjacent soil layers when applying appropriate Green's functions. The only disadvantage of this method is that it doesn't take into account the longitudinal potential attenuation caused by the pipeline metal resistivity. This disadvantage is especially emphasized in the case of protection of very long pipelines [14]. Therefore, in such situations it is suitable to use a hybrid boundary element/finite element (BEM/FEM) method [14-18].

This paper presents the mathematical mode based on the solution of integral field equations and method of superposition for calculation of metallic pipeline polarization. This model takes into account electric potential attenuation along the pipeline.

2. MATHEMATICAL MODEL

As previously mentioned, the purpose of CP system modelling is to determine potential distribution (level of polarization) on the protected object surface. Due to the complexity of the system, calculations of the system parameters are usually carried out on the simplified models with appropriate approximations, in order to simplify the mathematical model. Mathematical model given in this paper is based on the following assumptions:

- dimension of the anode groundbed is much smaller in comparison to the length of the protected pipeline, therefore it can be modelled as the point current sources,
- diameter of the protected pipeline is significantly smaller compared to its length, therefore it can be modeled as a line current source.

Geometry of impressed current cathodic protection (ICCP) system of the pipeline is shown on Fig. 1 and Fig. 2. The geometry of the system and established the symmetry of the analyse system, indicating that the calculation of the relevant parameters (protected pipeline polarization) should be carried out in a cylindrical coordinate system (r, θ, z). In the analysed system, anode groundbed and drainage point are placed on the following coordinates:

- anode groundbed: $r = h$; $\theta = 0$; $z = 0$
- drainage point: $r = r_2$; $\theta = 0$; $z = 0$

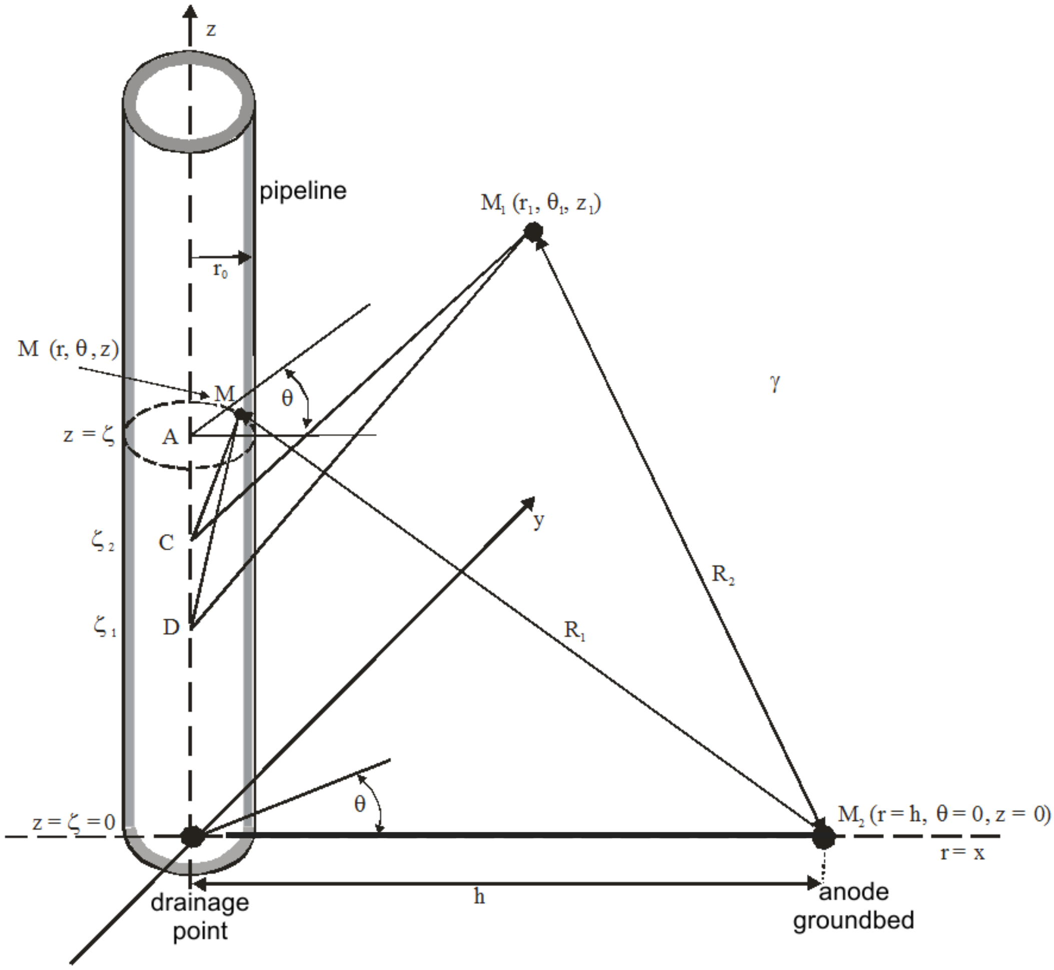

Fig.1. The geometry of ICCP system of pipeline

When calculating the potential distribution in the electrolyte, three current sources must be taken into account. First there is point a current source that represents the anode groundbed, second is the line current source that represents the pipeline and it is caused by induction from the anode groundbed, the third is also line current source which represents the impact of the drainage zone. The value of the potential at arbitrary point M_1 in the electrolyte caused by current sources in the system can be calculate using the following relation [5,6]:

$$\varphi_{el} = \frac{1}{4\pi\gamma_{el}}\left[\frac{I}{\sqrt{h^2+r^2-2hr\cos\theta+z^2}} + \int_{-\infty}^{+\infty}\frac{q(\zeta)d\zeta}{\sqrt{r^2+(z-\zeta)^2}} + \int_{-\infty}^{+\infty}\frac{q'(\zeta)d\zeta}{\sqrt{r^2+(z-\zeta)^2}}\right] \quad (1)$$

where: γ_{el} - conductivity of the electrolyte (S/m), I – protection current intensity (A), h - distance between anode groundbed and the centre of pipeline (m), r, θ, z – coordinates of the cylindrical coordinate system, ζ - distance of arbitrary point M in the direction of the axis-z (m), $q(\zeta)$ – line current density (A/m).

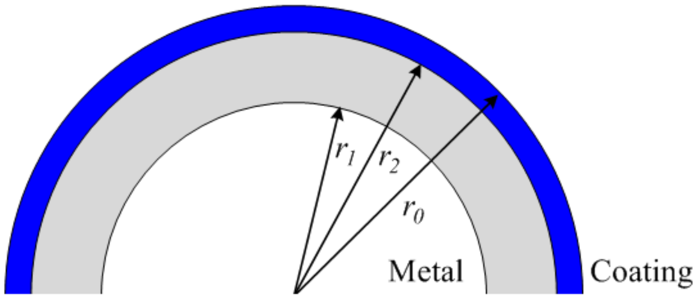

Fig.2. The geometry of pipeline

As for the calculation of pipeline polarization it is necessary to have value of the potential close to pipeline ie. for $r = r_0$, then the (1) can be simplified and takes the following form:

$$\varphi_{el} = \frac{1}{4\pi\gamma_{el}}\left[\frac{I}{\sqrt{h^2+z^2}} + \int_{-\infty}^{+\infty}\frac{q(\zeta)d\zeta}{\sqrt{r_o^2+(z-\zeta)^2}} + \int_{-\infty}^{+\infty}\frac{q'(\zeta)d\zeta}{\sqrt{r_0^2+(z-\zeta)^2}}\right] \quad (2)$$

For determination of the pipeline polarization relevant points are on the metal surface ($r = r_2$) and on the surface of the coating ($r = r_0$). The potential difference between these two points, without taking the value of the stationary potential, represents the polarization of the pipeline. Polarization of the cathodically protected pipeline can be calculated by following equation [5,6,11,13]:

$$\psi = \varphi_{el} - \varphi_M \quad (3)$$

where ψ is pipeline polarization (V), φ_M pipeline metal potential (V) and φ_{el} is potential on interference coating/electrolyte (V).

Pipeline coating can be considered as the highly resistant conductor, therefore current density that is flowing through pipeline coating can be calculated by using following relation:

$$\sigma = \frac{\varphi_{el} - \varphi_M}{R'} \quad (4)$$

where σ is current density flowing through the coating (A/m) and R' is resistivity of the coating (Ω/m).

This current density is equal to the current density emerging from the electrolyte, so that the boundary condition is valid:

$$\left(\gamma_{el}\frac{\partial\varphi_{el}}{\partial r}\right)_{r=r_0} = \frac{\sigma}{2\pi r_0} \quad (5)$$

By combining two previous equations (4) and (5), it can be written:

$$\left(\gamma_{el}\frac{\partial\varphi_{el}}{\partial r}\right)_{r=r_0} = \frac{\varphi_{el} - \varphi_M}{2\pi r_0 R'} \quad (6)$$

Potential gradient in the electrolyte can be determined from equations 2 in the form:

$$\left(\frac{\partial \varphi_{el}}{\partial r}\right)_{r=r_0} = -\frac{r_0}{4\pi\gamma_{el}}\left\{\int_{-\infty}^{+\infty}\frac{q(\zeta)\,d\zeta}{\left[r_0^2+(z-\zeta)^2\right]^{3/2}} + \int_{-\infty}^{+\infty}\frac{q'(\zeta)\,d\zeta}{\left[r_0^2+(z-\zeta)^2\right]^{3/2}}\right\} \tag{7}$$

Then it can be written:

$$-\frac{r_0}{2}\left\{\int_{-\infty}^{+\infty}\frac{q(\zeta)\,d\zeta}{\left[r_0^2+(z-\zeta)^2\right]^{3/2}} + \int_{-\infty}^{+\infty}\frac{q'(\zeta)\,d\zeta}{\left[r_0^2+(z-\zeta)^2\right]^{3/2}}\right\} = \frac{\varphi_{el}-\varphi_M}{R'} \tag{8}$$

As already indicated φ_M represents metal pipeline potential. The current density through the pipelines coating can be represented by the following equation:

$$\gamma_M \frac{d^2\varphi_M}{\mathrm{d}z^2} = \frac{\varphi_M-\varphi_{el}}{R'} \tag{9}$$

Potential gradient in the pipelines metal along the *z*-axis is caused by current of the draining point I'. This current is equal to the current of the anode groundbed, and it is assumed that the current is distributed equally on both sides of pipeline from point drainage. Therefore, potential gradient on the metal surface can be calculated from following equation:

$$\left(\frac{d\varphi_M}{dz}\right)_{z=0} = -\frac{I'}{2\gamma_M} \tag{10}$$

When calculating polarization of cathodically protected pipeline, is suitable to separate (2) into two components wherein the first two terms on the right side of equation represent the component φ'_{el} while the third term represent component φ''_{el}. Then pipeline polarization can be calculated as follows:

$$\psi = \psi' + \psi'' \tag{11}$$

2.1 Polarization component ψ′

The initial system of equations for determining the expression for the polarization components ψ' is:

$$\varphi'_{el}(r=r_0) = \frac{1}{4\pi\gamma_{el}}\left(\frac{I}{\sqrt{h^2+z^2}} + \int_{-\infty}^{\infty}\frac{q(\zeta)\,d\zeta}{\sqrt{r_0^2+(z-\zeta)^2}}\right) \tag{12}$$

$$-\frac{r_0^2}{2}\int_{-\infty}^{\infty}\frac{q(\zeta)d\zeta}{\left[(z-\zeta)^2+r_0^2\right]^{3/2}} = \frac{\varphi'_{el}-\varphi'_M}{R'} \tag{13}$$

$$\gamma_M \frac{d^2\varphi'_M}{\mathrm{d}z^2} = \frac{\varphi'_M-\varphi'_{el}}{R'} \tag{14}$$

Previous system of equations can be easily solved by using the Fourier transformation. After application of the Fourier transformation, system of equation can be rewritten in the following form:

$$\overline{\varphi'_{el}} = \frac{1}{4\pi\gamma_{el}}\left[\bar{q}\,2K_0(r_0 t) + 2IK_0(ht)\right] \tag{15}$$

$$-\frac{r_0^2}{2}\bar{q}\frac{2t}{r_0}K_1(r_0 t) = \frac{\overline{\varphi'_{el}} - \overline{\varphi'_M}}{R'} \tag{16}$$

$$-t^2\gamma_M\,\overline{\varphi'_M} = \frac{\overline{\varphi'_M} - \overline{\varphi'_{el}}}{R'} \tag{17}$$

where $K_0(r_0t)$, $K_0(ht)$ and $K_1(r_0t)$ are Bessel's functions of imaginary argument, t is the transform variable of z.

From (17) follows:

$$\overline{\varphi'_M}\,(1 + R'\gamma_M t^2) = \overline{\varphi'_{el}} \tag{18}$$

From (16) expression for the transformed current density can be obtained in the form:

$$\bar{q} = -t\cdot\gamma_M\,\frac{1}{r_0K_1(r_0t)}\overline{\varphi'_M} \tag{19}$$

By combination equations (15), (18) and (19) expression for transformed potential of metal surface can be obtained in following form:

$$\overline{\varphi'_M} = \frac{I}{2\pi\gamma_M}\,\frac{K_0(ht)}{\dfrac{\gamma_{el}}{\gamma_M} + R'\gamma_{el}t^2 + \dfrac{tK_0(r_0t)}{2\pi r_0\,K_1(r_0t)}} \tag{20}$$

From (18) and (20), expression for the transformed potential in the electrolyte near the surface of the pipeline can be written in the form:

$$\overline{\varphi'_{el}} = \frac{I}{2\pi}\,\frac{K_0(ht)\left(R't^2 + \dfrac{1}{\gamma_M}\right)}{\dfrac{\gamma_{el}}{\gamma_M} + R'\gamma_{el}t^2 + \dfrac{tK_0(r_0t)}{2\pi r_0\,K_1(r_0t)}} \tag{21}$$

By applying the inverse Fourier transformation, the expression for calculation of polarization can be written:

$$\psi'(z,h) = \frac{2}{\pi}\int_0^{\infty}\overline{\varphi'_{el}} - \overline{\varphi'_M}\cos tz\,dt \tag{22}$$

After the inclusion of the expressions for transformed potential of metal surface and transformed potential in the electrolyte, polarization component ψ' can be calculated by using following equation:

$$\psi'(z,h) = \frac{IR'}{\pi}\int_0^\infty \frac{\frac{t^2}{\pi}K_0(ht)}{\frac{\gamma_{el}}{\gamma_M} + R'\gamma_{el}\,t^2 + \frac{tK_0(r_0 t)}{2\pi r_0\, K_1(r_0 t)}}\cos tz\, dt \tag{23}$$

2.2 Polarization component ψ''

Second component of polarization ψ'' is caused by drainage zone. Expression for calculation of this component of polarization is performed from the following system of equations:

$$\varphi''_{el}(r = r_0) = \frac{1}{4\pi\gamma_{el}}\int_{-\infty}^{\infty}\frac{q'(\xi)d\xi}{\sqrt{(z-\xi)^2 + r_0^2}} \tag{24}$$

$$-\frac{r_0^2}{2}\int_{-\infty}^{\infty}\frac{q'(\xi)d\xi}{\left[(z-\xi)^2 + r_0^2\right]^{3/2}} = \frac{\varphi''_{el} - \varphi''_M}{R'} \tag{25}$$

$$\gamma_M\frac{d^2\varphi''_M}{dz^2} = \frac{\varphi''_M - \varphi''_{el}}{R'} \tag{26}$$

$$\left(\frac{d\varphi_M}{dz}\right)_{z=0} = -\frac{I'}{2\gamma_M} \tag{27}$$

An analogous proceeding of solving as in the previous case can be applied to the given system of equation to determine the second component of polarization ψ''. This component of polarization can be determined by using following relation [10,11]:

$$\psi''(z) = -\frac{2I'R'}{\pi}\int_0^\infty \frac{\cos tz\, dt}{1 + t^2R'\gamma_M + \frac{\gamma_M R' tK_0(r_0 t)}{2\pi r_0 K_1(r_0 t)}} \tag{28}$$

2.3 Total polarization

Current in cathodic protection system is conserved, therefore it can be written:

$$I + I' = 0 \tag{29}$$

The total value of the pipelines polarization is obtained by applying the principle of superposition on the two predetermined polarization components:

$$\psi = \frac{I \cdot R'}{\pi}\left[\int_0^{\infty} \frac{\frac{t^2}{\pi}K_0(ht)}{\frac{\gamma_{el}}{\gamma_M} + R'\gamma_{el}t^2 + \frac{tK_0(r_0 t)}{2\pi r_0 K_1(r_0 t)}}\cos tz\, dt + 2\int_0^{\infty}\frac{\cos tz\, dt}{1 + t^2 R'\gamma_M + \frac{\gamma_M R' t K_0(r_0 t)}{2\pi r_0 K_1(r_0 t)}}\right] \tag{30}$$

3. CASE STUDY

This section presents the results of the calculation obtained by using previously presented mathematical model for ICCP system of the pipeline with different values of the input data. Model was implemented by using Matlab software package. Below are given calculation results of the cathodic protection system for the different value of the electrical resistivity of electrolyte (soil) and different coating insulation resistivity. The input parameters that are used in all calculations as constant parameters are listed in Tables I.

Table I: Input parameters for calculations

Parameter	**Value**
Current	1 (A)
Anode groundbed distance	100 (m)
Resistance of pipeline metal	0,135 (Ωmm^2/m)
Pipeline diameter	600 (mm)

3.1 Case study 1 (Impact of the soil resistivity)

In order to analyse the impact of soil resistivity on the polarization of the cathodically protected pipeline, authors calculated the individual components of pipeline polarization for corrosive aggressive soil (ρ_{el} = 50 Ωm), moderately aggressive soil (ρ_{el} = 200 Ωm) and soil that is not corrosive aggressive (ρ_{el} = 500 Ωm). Calculations were performed for coating resistivity of R = 5000 Ωm. Calculation results are presented on Fig. 3, Fig. 4 and Fig. 5.

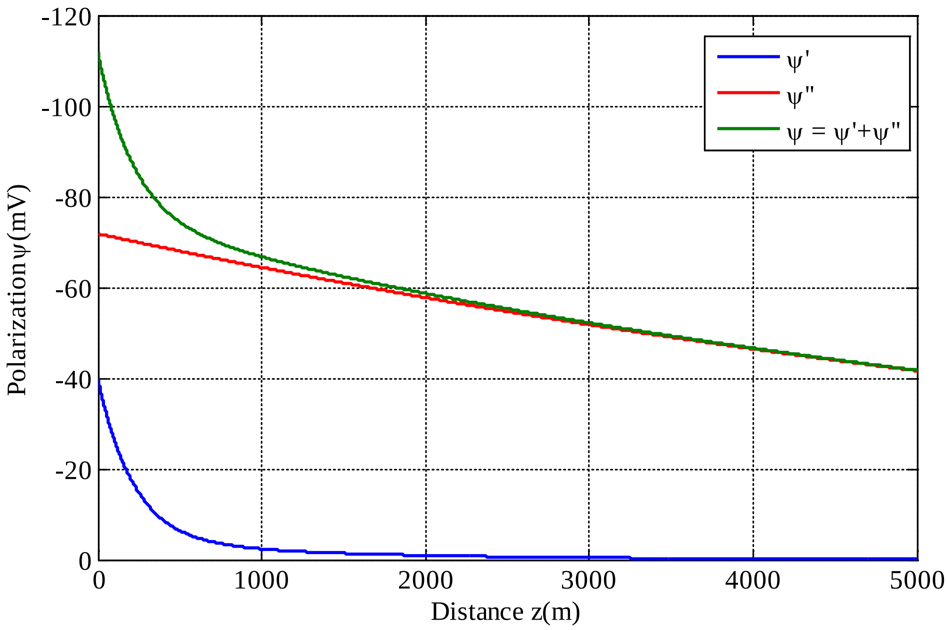

Fig.3. The change of polarization along the pipeline for ρ_{el} = 50 Ωm

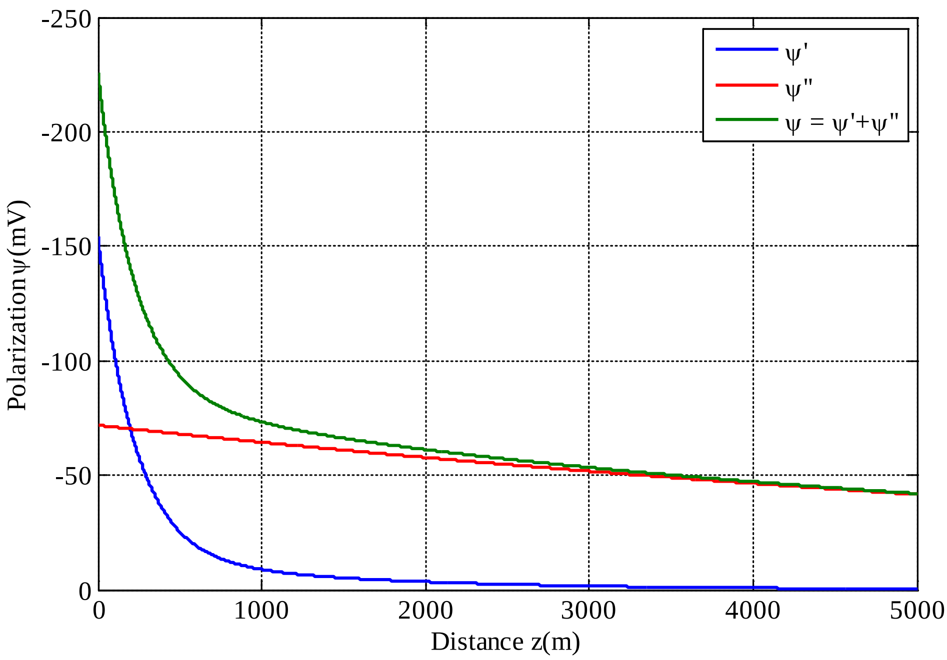

Fig.4. The change of polarization along the pipeline for ρ_{el} = 200 Ωm

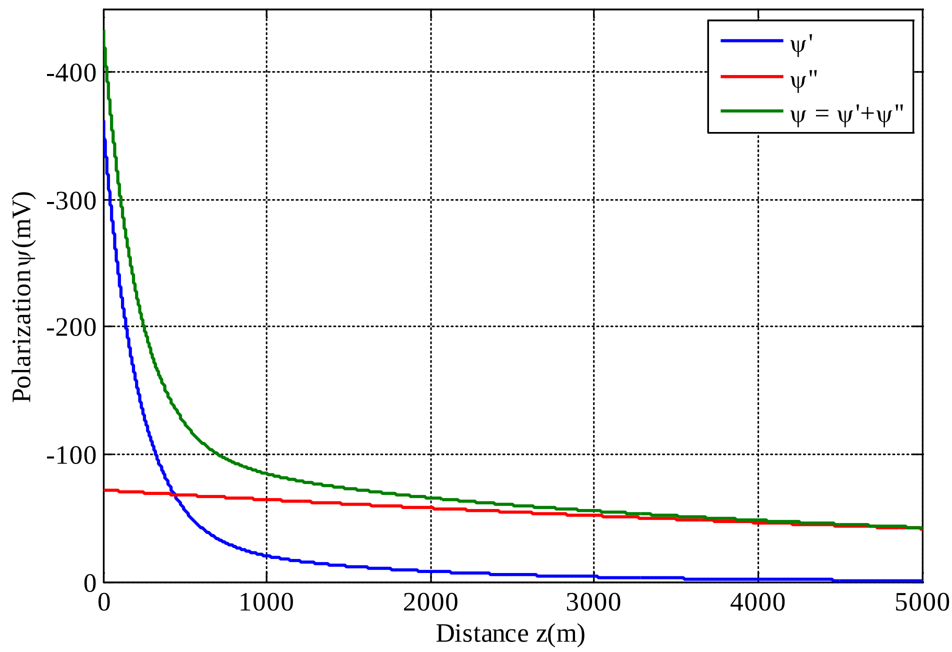

Fig.5. The change of polarization along the pipeline for ρ_{el} = 500 Ωm

From calculation results given on the Fig. 3, Fig 4 and Fig. 5 it can be noted that change of the soil resistivity has a significant impact on the polarization component ψ', while impact of the soil resistivity on the polarization component ψ'' can be neglected. From given results it is evident that the increases of the soil resistivity cause the higher value components of polarization ψ' and consequently the total polarization ψ. The reason for this lies in the fact that the potential distribution caused by anodic groundbed is directly proportional to surrounding soil resistivity.

3.2 Case study 2 (Impact of the coating resistivity)

Another important factor for pipeline protection from corrosion is the coating resistivity. In this section, impact of the pipelines coating insulation resistivity on polarization components will be analysed. Calculations were performed for cases of poor coating with numerous damage (R = 200 Ωm), coating of satisfactory quality with few defects (R = 2000 Ωm) and high-quality coating with minor damage (R = 5000 Ωm). In calculations of pipeline polarization components it was that system is placed in the corrosive aggressive soil with soil resistivity ρ_{el} = 50 Ωm. Calculation results are presented on Fig. 6, Fig. 7 and Fig. 8.

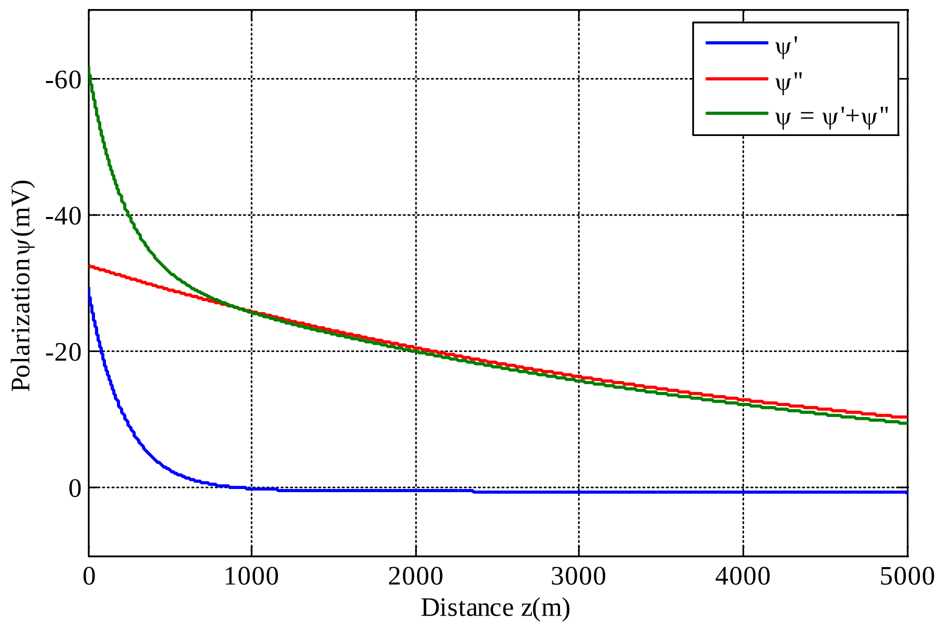

Fig.6. The change of polarization along the pipeline for R = 200 Ωm

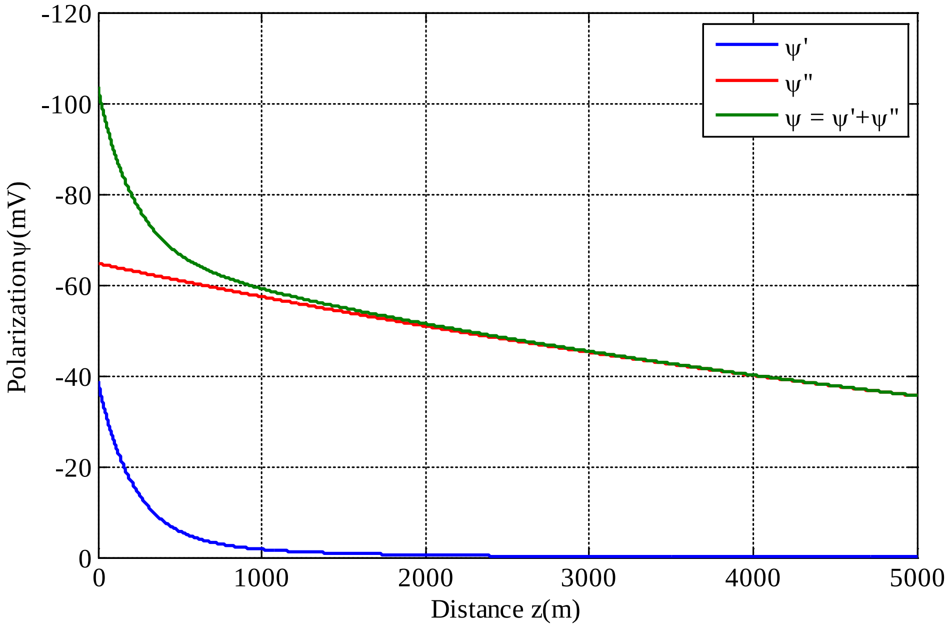

Fig.7. The change of polarization along the pipeline for R = 2000 Ωm

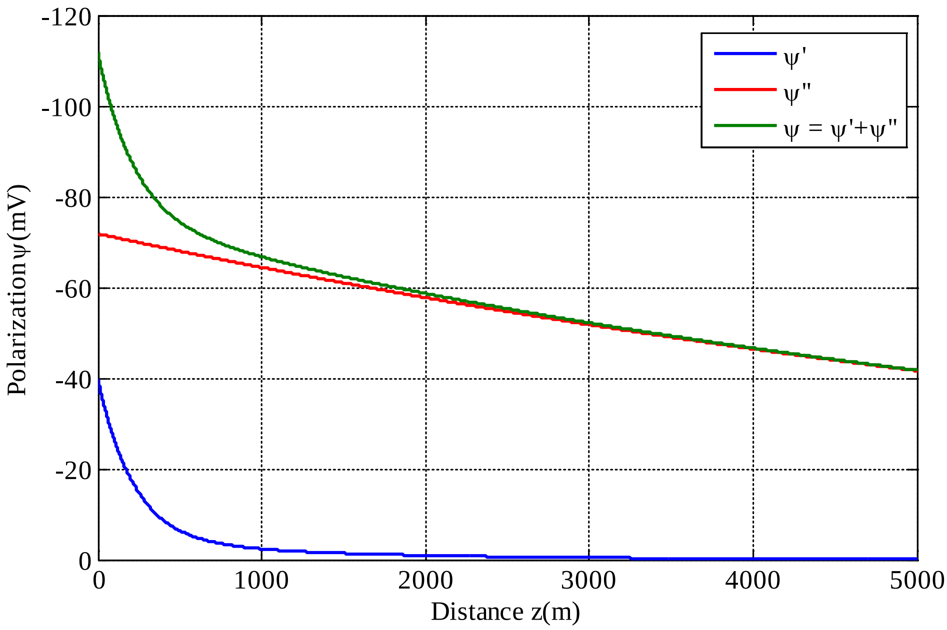

Fig.7. The change of polarization along the pipeline for R = 5000 Ωm

From calculation results given on the Fig. 6, Fig 7 and Fig. 8 it can be noted that change of the coating resistivity have impact on the both components of polarization ψ' and ψ''. It is evident that the polarization component ψ' is lower at higher values of coating resistivity. The reason for this is that increment in the coating resistivity is lowering down induced current in the pipeline. Also, from the results it is evident that the value of the components of polarization ψ'', which depends on the quality of pipeline insulation, is much greater for higher value of coating resistivity. This component of polarization is dominant in total value of polarization ψ and very important fact is that total polarization decreases very slowly with increasing distance from the drainage point for high value of the coating resistivity.

4. CONCLUSION

The main task in determining the efficient cathodic protection system is to define technically correct level of polarization of protected object. This paper presented the mathematical model based on integral equations for calculation of the polarization of the cathodicaly protected pipelines. Given mathematical model is particularly suitable for calculation polarization of long underground pipelines (coated or uncoated) which are protected with impressed current cathodic protection systems.

The presented mathematical model was used for analysis of impact of different parameters on the level of polarization of pipelines. The influence of soil resistivity and coating resistivity to the individual components and the total polarization of the pipeline has been analyzed by using presented model. Knowing of the individual components of polarization as well as total polarization of the chatodicaly protected pipeline is essential for the selection of optimal protective methods as well as disposition of protection system equipment.

5. ACKNOWLEDGEMENTS

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