# ELECTRIC POWER SYSTEM OPERATION – STEADY-STATE VOLTAGE STABILITY

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## **ABSTRACT**

There are many software possibilities capable to solve demanding power engineering problems. Continuation load flow analysis is shown along with Author's developed computational software in Matlab envi-ronment for full-scale voltage stability analysis of larger power systems. Steady-state system stability is defined as the capability of the network to withstand a small disturbance (fault occur-rence, small change of parameters, topology modification) without leaving its stable equilibrium point.

For comparison, alternative approach is further introduced for voltage stability solution using the non-commercial optimization tool. This paper deals with steady-state voltage stability prob-lem as one of the main topics of interest in today's power system operation and control worldwide.

## 1. INTRODUCTION

Steady-state system stability is defined as the capability of the network to withstand a small disturbance (fault occur-rence, small change of parameters, topology modification) without leaving its stable equilibrium point. With increasing network loading (consumption), bus voltages slowly decrease due to insufficient var reserve until significant voltage drop appears (voltage collapse). Such pathological situation, oc-curring in a relatively large time frame (from several seconds to tens of minutes even hours) may be caused also by long electrical distances between reactive power sources and loads, low source voltages, crucial changes in network topol-ogy and insufficient level of var compensation.

In this paper, static voltage stability problem has been introduced along with two possible approaches for its comprehensive solving - using the NEOS Server for Optimization and developed computational software in Matlab environment. As the results, level or reactive power reserve with VSMI and VSF values has been computed for understanding voltage behaviour in the system under higher loading conditions.

Therefore, voltage stability has been considered as the top-priority topic for avoiding related black-out/islanding problems, which the public witnessed especially during the last two decades.

Although the voltage stability is a dynamic problem, it can be suitably analyzed using static methods. Therefore, the symbolic-complex method can be used for analytical solution

using standard load flow equations. In this case, final V-P, P- $\theta$  and V- $\theta$  dependencies can be derived. After calculating the derivatives of former two formulas (with respect to  $\lambda$  and  $\theta$ ), values of  $v_{crit}$  and  $\lambda_{max}$  can be obtained. Unfortunately, this approach can be used only for 2-bus and simple 3-bus networks since otherwise the load flow equations become strongly nonlinear and cannot be solved analytically.

Both usually applied numerical methods for the load flow analysis (the Gauss-Seidel and the Newton-Raphson) can be used for reliable drawing of the curve's stable part only. When approaching the bifurcation point, both of these meth-ods fail to converge since the Jacobian is becoming singular (det  $J \approx 0$ ). For solving the voltage stability problem using the traditional numerical methods, the Gauss-Seidel is often preferred due to broader range of numerical stability. On the other hand, approximate solutions of the singular point are obtained in usually long computation times. Because of CPU requirements, several modifications of the Newton-Raphson were developed and tested in the past for assessing voltage stability and collapse scenarios of electric power system

To simplify design layout, automate verification of all rules and limitations and minimize mistakes computer application was developed to solve these tasks.

Above described problem can be graphically expressed by the V-P curve (also referred to as "nose curve"). Its peak, called "saddle node bifurcation point", is determined by maximum loadability factor  $\lambda_{max}$  (x-axis) and critical voltage vcrit (y-axis). From this point, two parts of the V-P curve are built - the upper (stable) and the lower (unstable). Nevertheless, current position of the system operating point on the V-P curve is not known along with its distance from the voltage collapse (i.e. voltage stability margin).

## 2. CONTINUATION LOAD FLOAD ANALYSIS

Suitably modifies conventional load flow equations to be-come stable also in the bifurcation point and therefore to be capable to draw both upper and lower parts of the V-P curve. To make it possible, it uses a two-step predictor/corrector algorithm along with single new unknown state variable (so-called "continuation parameter").

Predictor is a tangent extrapolation of the current opera-tion point estimating approximate state variable values in the new step - see Eqn. 1. As the continuation parameter, state variable with the highest change rate must be chosen (i.e.  $\lambda$  and V in flat and steep parts of the V-P curve, respectively). Step size must be carefully selected to speed-up the calculation and avoid convergence problems. When the process starts diverging, parameter  $\sigma$  must be decreased or the change of continuation parameter must be performed.

$$\begin{bmatrix} \theta \\ V \\ \lambda \end{bmatrix}^{predicted} = \begin{bmatrix} \theta_0 \\ V_0 \\ \lambda_0 \end{bmatrix} + \sigma \begin{bmatrix} \vdots & \vdots & K \\ & \vdots & K \\ & & \vdots \\ & & \vdots \\ & & & e_k \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$(1)$$

where:

K - vector of base-case active/reactive power loads  $\theta_0$ ,  $V_0$ ,  $\lambda_0$  - state variables from previous corrector step

 $J_{LF}$  - newly computed Jacobian

 $e_k$  - zero vector with modifications for elements re-lated to the continuation

parameter according to the position on the V-P curve

 $\sigma$  - step size for changing the continuation parameter value for the current predictor step

As the corrector, original Newton-Raphson is used for correcting state variables from the predictor step to satisfy load flow equations. Due to one new unknown parameter (loadability factor  $\lambda$ ), additional equation (Eqn. 2) must be included maintaining the value of the continuation parameter constant in the corrector step. This condition makes the final set of equations non-singular at the bifurcation point.

$$x_{k} - x_{k}^{predicted} = 0 (2)$$

Entire numerical process can be graphically demonstrated (see Fig. 1), where horizontal and vertical corrections are performed with respect to chosen continuation parameter.

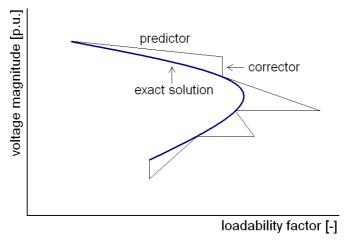


Fig. 1 Predictor/Corrector steps for CLF analysis

For voltage and var control, it is not possible to automatically presume that all PV buses will be switched to PQ when searching for the bifurcation point. Some buses may be eventually switched to PQ, which significantly modifies the shape of the V-P curve. However, many buses still may preserve regulation ability due to broad var limits or low local network transfers of reactive power.

Presented algorithm is adaptable for increasing the load in one or more PQ buses of the network. Increase of only active power load as well as of both active and reactive power loads with constant power factor can be realised.

When computing the V-P curve up to its bifurcation point, voltage stability margin index (VSMI) is computed for each bus - see Eqn. 3.

$$VSMI_{i} = 100 \left( V_{i(\lambda=1)} - V_{i(\lambda=\lambda_{max})} \right) / V_{i(\lambda=\lambda_{max})} [\%]$$
(3)

According to Eqn. 4, level of reactive power reserve can be also assessed at each point of the V-P curve.

$$Q_{reserve} = \sum_{i}^{PV} \min \{Q_i - Q_i^{\min}, Q_i^{\max} - Q_i\} [p.u.]$$

$$(4)$$

In each predictor step, dV elements are useful for identify-ing "weak buses", i.e. buses with large voltage variations in response to load change [2]. Eqn. 5 shows the formula for so-called "voltage sensitivity factor VSF". Certain amount of highest VSFs represents weak buses and areas of the network.

$$VSF_i = dV_i / \sum_{k=1}^n dV_k [-]$$
(5)

Identification of saddle nodes and Hopf bifurcations is comprehensively studied in [3]. More robust approach using energy functions for solving the voltage stability issue with deeper insight into large disturbances is presented in [4]. Further inventions to step-size modifications during the CLF process along with Q-limit guided CLF are introduced in [5].

# 2.1 NEOS - Server For Optimization

As an alternative approach, one particular non-commercial software package - the NEOS (Network Enabled Optimization System) Server [6] - has been also used for the voltage stability analysis.

Despite of free use of all NEOS solvers, its calculation time and memory restrictions must be taken into account. Regardless, the NEOS project is very robust programming tool and its free use makes it even more attractive for many different research areas.

# 2.2 Optimization Problem

minimize f(x) subject to

$$g(x) = 0; (6)$$

 $h_l \le h(x) \le h_u;$  $x_l \le x \le x_u.$  Function f(x) is an objective function, whose value has to be optimized. Function g(x) represents active and reactive power balance equations for each bus while all remaining equations (i.e. branch power flows, total power losses and shunt power flows from compensation units) are included in function h(x). Unknown vector x comprises voltage magni-tudes and angles in all buses of the system.

Prior to the structure (6), all input parameters and vari-ables must be fully specified corresponding to given con-stants and unknowns of the conventionally formulated load flow problem. Note: Switching of some PV buses to PQ is realized by using additional binary variables to express the exceeding of upper/lower var limits for each PV bus. As an objective function, the sum of all PV-to-PQ-switched PV buses must be minimized to hold voltage and var control ability. For complete description of each voltage stability prob-lem, large number of inputs must be specified. Therefore, specialized program has been created for converting the load flow input data structure from Matlab to AMPL format.

# 2.3 Voltage Stability - Case Studies - Analysis of a Larger Power System

The IEEE 300-bus test power system has been chosen for performing complete voltage stability study using developed computational software in Matlab environment.

First, the VSMI values for each non-slack bus of exam-ined network have been calculated - see Fig. 2 below. As visible, the majority of buses are relatively stable (by varying their voltages in range of 10 percent only). On the other hand, possible abnormal operations of the network cannot be as-sessed based on their voltage values.

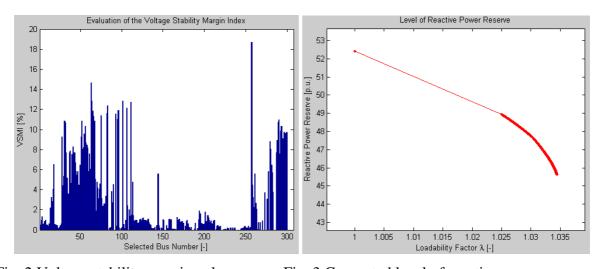


Fig. 2 Voltage stability margin values

Fig. 3 Computed level of reactive power reserve

Second, reactive power reserve in individual parts of the V-P curve has been projected - see Fig. 3.

In this case, the var reserve has been decreased by approx. 13 percent and became stable on relatively high values distant from zero. This was caused by very broad var limits in several PV buses of the system. It remains to say in this moment that the maximum loadability

factor  $\lambda_{max}$  was found with value of approx. 1.0344 [-]. However in practice, the maximum loadability factor should incorporate the viable bus voltage values (i.e. the values within their permitted voltage limits:  $\pm 5\%/\pm 10\%$  of  $V_n$ ). When taking this into account, the maximum loadability factor is only 1.0279 [-].

## 3. FINALLY AND CONCLUSION

To provide complete picture about the network, voltage sensitivity study has been performed to make and show critical areas where possible protective measures must be taken for pre-venting/minimizing negative effects of low-voltage system operations. Therefore, predictor values computed in the bifur-cation point have been taken and certain percentage of corre-sponding non-slack buses (i.e. with highest voltage-load sensitivities). Maximum sensitivity value was not higher than 0.366 %. Therefore due to relatively stable bus voltage behaviour, no remedial actions have to be done in this examined network. Otherwise, var compensation using shunt capacitors, SVCs and synchronous condensers or shedding of low-priority loads should be applied when approaching critical system operational stage.

In this paper, static voltage stability problem has been introduced. Although the employed CLF algorithm proved to be very reliable and robust for performing this type of analysis, the step-size calculation has not been fully investigated for opti-mizing the calculation speed. For this, many advanced meth-odologies can be applied (modified predictor, Q-limit guided CLF, etc.). Also, ill-conditioned problems do not pose much threat to CLF analysis. Therefore, thorough examination of these tasks will be performed in follow-up studies.

## 4. REFERENCES AND BIBLIOGRAPHY

- [1] Crow, M. Computational Methods for Electric Power Systems. CRC Press: 2002, ISBN: 0-8493-1352-X.
- [2] Kundur, P. Power System Stability and Control. McGraw-Hill: 1994, ISBN: 0-07-035958-X.
- [3] Chow, J.H., Wu, F.F. and Momoh, J.A. Applied Mathematics for Restructured Electric Power Systems Optimization, Control and Computational Intelligence. Springer: 2005, ISBN: 0-387-23470-5.
- [4] Dobson, I., Cutsem, T.V., Vournas, C., DeMarco, C.L., Venkatasubramanian, M., Overbye, T. and Canizares, C.A. Voltage Stability Assessment: Concepts, Practices and Tools Chapter 2. Power System Stability Sub-committee Special Publication by IEEE Power Engi-neering Society: 2002, ISBN: 0780378695.
- [5] Zhu, P. Performance Investigation of Voltage Stability Analysis Methods. PhD thesis, Brunel University: 2008.
- [6] http://neos.mcs.anl.gov/neos/
- [7] www.ee.washington.edu/research/pstca/
- [8] Veleba J., Irving M.: Load Flow Analysis with Voltage and Reactive Power Optimization in Distribution Networks, article- 9<sup>th</sup> International Conference, Advanced

- Methods in the Theory of Electrical Engineering, AMTEE 2009, WBU in Pilsen, Czech Republic, 2009, ISBN: 978-80-7043-821-3
- [9] Noháč K., Noháčová L.: Overview of today possibilities of computer simulation in power engineering, article- 6<sup>th</sup> International Conference "Control of power systems 2004", Štrbské Pleso, Slovak Republic, 16.-18. 6. 2004, Slovak University of Technology in Bratislava, ISBN: 80-227-2059-3, S 1-8

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