

## EFFICIENT CALCULATION OF EARTH FAULT CURRENTS IN COMPENSATED NETWORKS

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### ABSTRACT

*In this paper, a simplified approach for determining the ground fault currents is presented which is specifically useful in compensated systems. In this case, due to the estimation of the relative magnitudes of the equipment parameters, the equivalent circuit in symmetrical components of the single-pole fault is significantly reduced. Thus the electrical parameters of the impedances can be directly used.*

## 1. CALCULATION OF EARTH FAULT CURRENTS

### 1.1 Introduction

Phase to ground faults (earth faults) in medium voltage networks constitute the major part of fault events (see Fig. 1). Especially in view of the discussions concerning the so-called extinction limit, resp. the dangerous touch voltages occurring in the event of such an error, the determination of the fault current is very important.

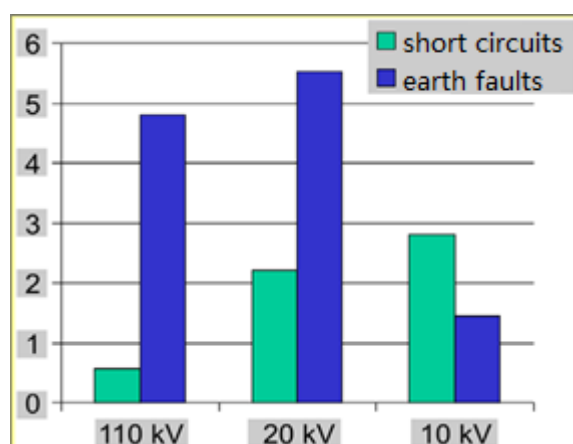


Fig. 1. Fault statistics for single and multi-phase faults, assigned to voltage levels, [1]

More recently, the question of the usefulness of operating earth fault compensated networks (with “arc extinction”) has been raised repeatedly in medium voltage networks<sup>1</sup>. As a result of the current state of discussion it can be stated that - apart from justified exceptions, such as aged cable networks - the maintenance of this operating regime is often useful. Especially in mixed networks, i.e. cable networks with a larger proportion of overhead lines, a change of the compensated neutral-point treatment would lead to a massive increase in the earth-fault current which has a deep impact on the grounding design of the substations etc. and might provoke necessarily upgrading actions of the plant grounding apparatus.

To assess the plant safety in the case of an earth fault calculations of the ground-fault current are required to assess the safety according to the valid standards.



Fig. 2. Standing arc in a substation switchgear

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<sup>1</sup> In the case of resonant grounded networks (with Petersen coil) the capacitive earth fault current is compensated in such a way that at the fault point the earth fault current as well as the resulting earth potential rise (EPR) are minimized by the arc suppression coil (Petersen coil). In most cases, this induces an automatic termination of the fault arc in overhead systems. Thus the necessity of switching off the faulted line is eliminated. The resonant grounded network can be operated continuously in the event of a single-pole fault and the reliability of supply of this network is not impaired.

## 1.2 Task

The calculation of the earth fault current is carried out mainly for two reasons:

- Determination of the residual current at the earth fault location (for example, to determine the earth potential rise (EPR) and the associated hazards)
- Calculation of the phasors for the zero sequence currents and the accompanying residual voltages at various points in an electrical network in order to determine the protection settings concerning their sensitivity and pick-up reliability correctly. This approach is often referred to as "ground fault engineering".

Usually the method of symmetrical components is used to accomplish these purposes. In practice, the use of network calculation programs for detailed analysis is recommended.

Experience shows that the use of symmetrical components requires a considerable amount of theoretical training. Because of the necessary matrix operations and the scaling factors it is often complicated and may lead in many cases to calculation errors.

## 1.3 Methodology

Based on the theory of symmetrical components, the equivalent circuit for a phase to ground fault is the series circuit of positive, negative, and zero sequence components (see Fig.3). It is simplified by neglecting the positive and the negative sequence system in order to calculate the earth fault current (ground fault in the earth fault compensated network).

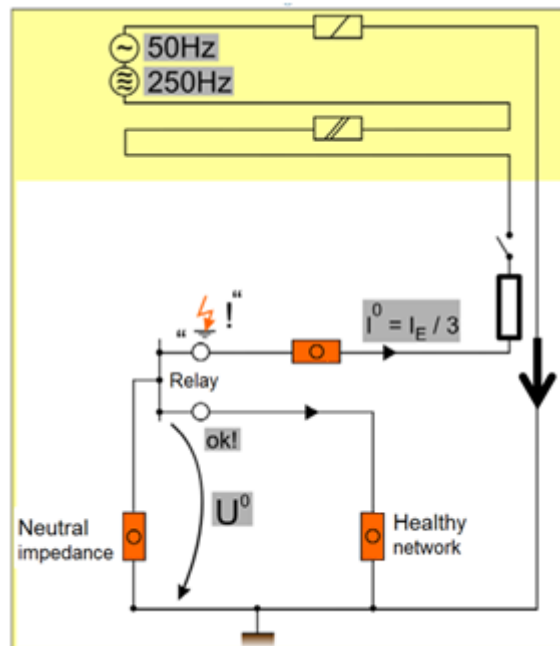


Fig. 3. Equivalent circuit for the single-pole fault with relevant parameters, [1]

The derivations are based on the application of the theory of symmetrical components. Accordingly the methodology is explained stepwise.

Calculations carried out for the purpose of simplification yield usually an accuracy of  $\pm 10\%$ . In the course of the following simplifications, the zero sequence impedances of any neutral star point transformer and the positive-sequence impedances of transformers or of the upstream high-voltage grid - expressed by the short-circuit power  $S_k$ ", as well as the angle of the earth return path factor (" $k_0$ ") are neglected.

**Comments regarding the simplifications:** By a numerical analysis of the current flow - represented by the symmetrical component system – it can be shown that the series connection of positive, negative, and zero sequence components can be interpreted as the series connection of the phase voltage of phase L1 – as a driving source voltage with an internal source impedance – with a high-ohmic burden. The positive and negative sequence system can be considered as the source impedance of the voltage, whereas the zero sequence system represents the burden to the source.

Considering typical numerical values, the series connection of positive and negative sequence takes up only a fraction of the total impedance (positive, negative and zero sequence) and can therefore be neglected.

**Comment on the following derivation steps:** Starting from a preliminary stage (considering the ground fault in a TN-system), the earth fault current is determined in a consecutive way for the following network types:

- a) Earth fault in a TN-system (preliminary stage)
- b) network with current limiting star point resistor
- c) network with isolated neutral
- d) network with neutral arc suppression coil (Petersen coil): fundamental and harmonic currents
- e) network with neutral arc suppression coil (Petersen coil) and short-term resistance grounding of the neutral point

At selected locations, the results are demonstrated by sample calculations. In each of the above steps, only those network elements are added which are necessary for understanding the methodology and which are necessary to obtain the required calculation accuracy.

#### 1.4 Calculation of the ground-fault in a TN system (preliminary stage)

As a preliminary stage a ground fault in a TN system (= short circuit) between a conductor and the PEN conductor is considered (see Fig.4 and Fig.5).

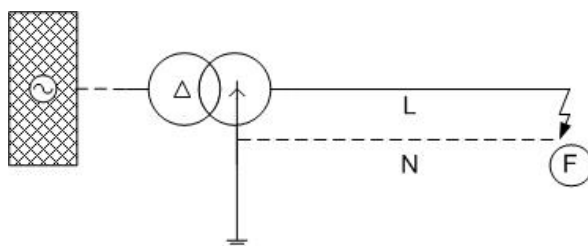


Fig. 4. Earth fault in a TN system: circuit diagram, F...fault location

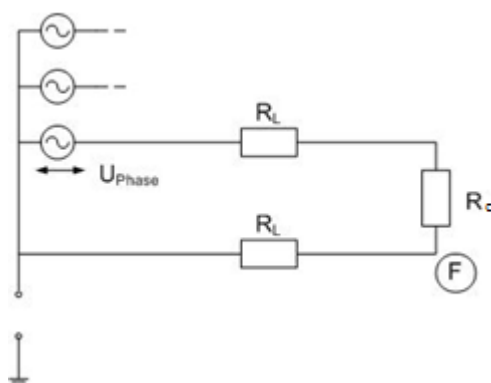


Fig. 5. Physical current flow during earth fault in a TN system, F...fault location

According to the illustration the earth fault current is given by:

$$I_{F, TN} = U_{Phase} / (R_L + R_c + R_L) = U_{Phase} / (2 \cdot R_L + R_c) \quad (1)$$

where:  $I_{F, TN}$  - earth fault current at the fault location,

$U_{Phase}$  - phase to ground voltage,

$R_L$  - resistance of the phase conductor,

$R_c$  - contact resistance at the fault location

When taking into account the influence of the inductive line impedance

$$I_F = U_{Phase} / (Z_L + R_c + Z_{PEN}) \approx U_{Phase} / (2 \cdot Z_L + R_c) \quad (2)$$

where:  $I_F$  - earth fault current,

$Z_L$  - impedance of the phase conductor,

$Z_{PEN}$  - impedance of the PEN conductor

This relationship can be simplified in approximation to the system of symmetrical components, neglecting the series connection of the positive and negative sequence (see Fig. 6).

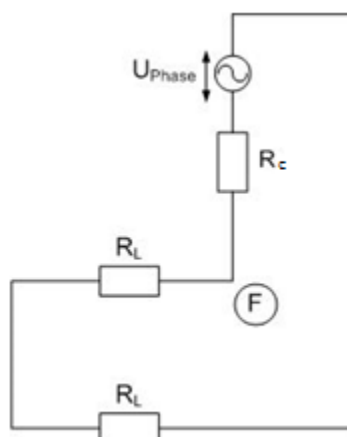


Fig. 6. Simplified representation of a ground fault in TN systems - equivalent circuit, F ... fault location

### 1.5 Calculation of the ground-fault current in a resistance grounded network

To answer the core question of the earth fault calculation "How is the current path of the earth fault current closed, while looking at the return part of the current loop from ground into the elsewhere highly galvanically isolated power network?" it is necessary to consider the following current loop (see Fig.7):

- substation busbar  $\diamond$  phase conductor  $Z_L$   $\diamond$  line to ground transition resistance  $R_c$   $\diamond$  earth return path impedance  $Z_m$   $\diamond$  star point resistance  $R_{Stp}$   $\diamond$  substation busbar

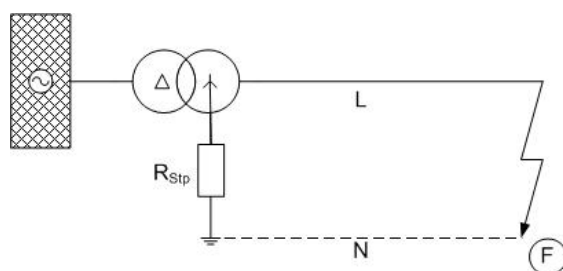


Fig. 7. Ground fault in a resistance-network – Electrical diagram; F ... fault location

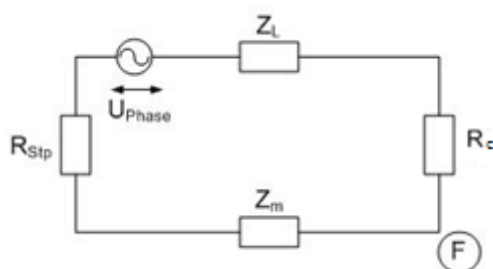


Fig. 8. Physical current flow on ground fault in a resistance-network; F ... fault location

For the relationship between the line impedance  $Z_L$  and the earth return circuit impedance  $Z_m$ , one can use the well-known context from the power system protection.

$$Z_m = k_0 \cdot Z_L \quad \text{with} \quad k_{0,\text{cable}} = 0,8 \quad k_{0,\text{OHL}} = 1 \quad (3)$$

In addition, it is permissible with good approximation to replace the circuit impedance  $Z_L$  by the line reactance  $X_L$ . As shown in Fig.9 the earth fault current  $I_F$  is given by (4), where  $U_N$  represents the nominal voltage,  $R_c$  is the contact resistance at the fault location and  $R_{Stp}$  equals the neutral point resistance of the considered circuit.

$$I_F = (U_N / \sqrt{3}) / \sqrt{((R_c + R_{Stp})^2 + [(1+k_0) \cdot X_L]^2)} \quad (4)$$

This value, as shown above, results also (in accordance with the system of symmetrical components) from the slightly redrawn representation shown in Fig. 9.

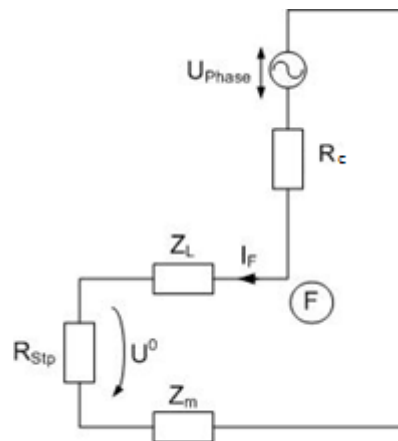


Fig. 9. Ground faults in a resistance grounded network - equivalent circuit; F ... fault location

The fault current is in practice determined by the type of earthing method essentially through the neutral point resistance  $R_{Stp}$ .

## 1.6 Calculation of the ground-fault current in a network with isolated neutral

To answer the a.m. core question "How is the current path of the earth fault current closed, while looking at the return part of the current loop from ground into the elsewhere highly galvanically isolated power network?" in this case the ground to wire capacitances have to be considered (see Fig.10).



Fig. 10. Arc and capacitive ground to wire current paths

In a network with galvanically isolated neutral the ground fault current returning from the fault location is divided into different sub-current paths ( $I_{F,1}$  to  $I_{F,n}$ ) by means of their respective single ground to wire capacitances (see Fig.11).

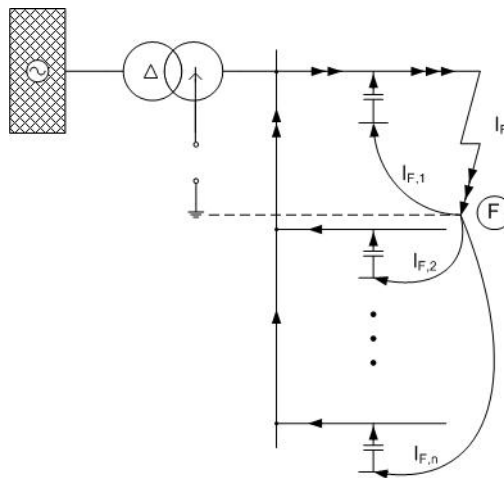


Fig. 11. Physical current flow in case of a ground fault in a network with isolated neutral; F ... fault location

If one subsumes these individual ground to wire capacitances to a total capacity  $X_{cap}$  as well as the individual sub-current paths to a total current ( $I_{F, CAP}$ ), one obtains the following schematic diagram (Fig.12).

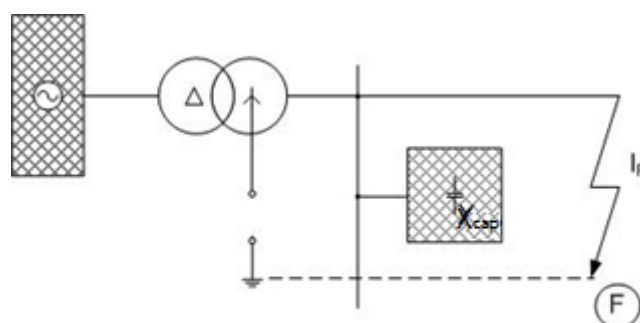


Fig. 12. Ground fault in a network with isolated neutral - electrical diagram; F ... fault location

As shown in figure 11 and 12, the earth fault current  $I_F$  consists of the capacitive current contributions of each network segment as follows:

In a cable network with the length  $l_{cable}$  each km of cable length contributes a ground fault current of about 3 A. 1 km of overhead line contributes approximately 1/30 of this value. As a result, the following capacitive earth fault current is flowing (with neglected line to ground transition resistance, worst-case):

$$I_F = I_{grid, cap, e/f} = l_{cable} \cdot i'_{cable} + l_{OHL} \cdot i'_{OHL} \quad (5)$$

- where:
- $U_{Phase}$  - phase to ground voltage,
  - $l_{cable}, l_{OHL}$  - Total length of the cables / overhead lines,
  - $i'_{cable}$  - capacitive current contribution per kilometer of cable
  - $i'_{OHL}$  - capacitive current contribution per kilometer of overhead line

This value, calculated according to (5), can also be derived (similar to the system of symmetrical components) from a slightly redrawn representation shown in Fig.13.

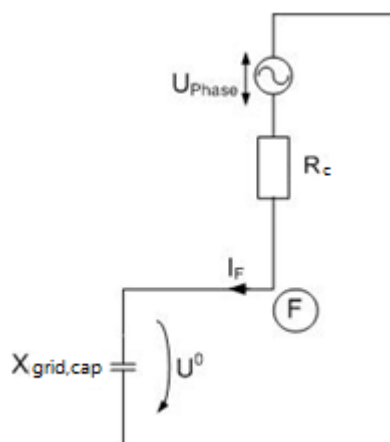


Fig. 13. Ground fault in a network with isolated neutral - equivalent circuit; F... fault location

According to Fig.13, the equivalent total network capacitance  $X_{\text{grid, cap}}$  for the fault location "F" is the internal impedance of the network.

$$X_{\text{grid, cap}} = U_{\text{Phase}} / I_{\text{grid, cap, e/f}} = U_{\text{Phase}} / (I_{\text{cable}} \cdot i'_{\text{cable}} + I_{\text{OHL}} \cdot i'_{\text{OHL}}) \quad (6)$$

**Remark:** In case the line to ground transition resistance cannot be neglected ( $R_c > \text{some } 100 \Omega$ ), the earth fault current according to Fig.13 has to be calculated from the complete current loop:

- Transformer  $\diamond$  line to ground transition resistance  $R_c \diamond$  equivalent total network capacitance  $X_{\text{grid, cap}}$

$$I_F = (U_N / \sqrt{3}) / \sqrt{(R_c^2 + X_{\text{grid, cap}}^2)} \quad (7)$$

### 1.7 Calculation of the ground-fault current in a network with resonant grounding – calculation of the 50-Hz-components

When answering again the a.m. core question "How is the current path of the earth fault current closed, while looking at the return part of the current loop from ground into the elsewhere highly galvanically isolated power network?" for this case, additionally to the ground to wire capacitances the newly created current return path through the arc suppression coil (Petersen coil) has to be considered (see Fig.14).

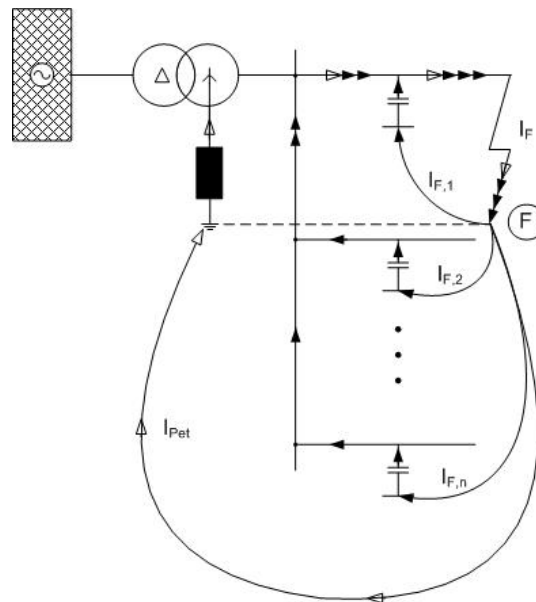


Fig. 14. Physical current flow for a ground fault in a network with earth fault compensation; F ... fault location

In an analogous way to the procedure for a network with isolated neutral the following circuit diagram (Fig.15) can be derived:

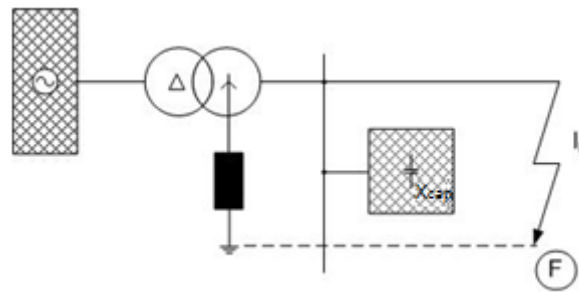


Fig. 15. Ground fault in a network with earth fault compensation; F ... fault location

As shown in Fig.14 and Fig.15, the earth fault current  $I_F$  is again obtained by the superposition of the capacitive current contributions of the individual sections of the network ( $I_{\text{grid, cap, e/f}}$ ), but this time it is also superimposed - with opposite sign - with the inductive coil current  $I_{\text{Pet}}$ . As a result, the following total ground fault current is obtained (with line to ground transition resistance at the fault location):

$$I_F = I_{\text{Pet}} - I_{\text{grid, cap, e/f}} = U_{\text{Phase}} / X_{\text{Pet}} - (I_{\text{cable}} \cdot i'_{\text{cable}} + I_{\text{OHL}} \cdot i'_{\text{OHL}}) = m \cdot I_{\text{grid, cap, e/f}} \quad (8)$$

where:  $I_{\text{Pet}}$  - current through the arc suppression coil (Petersen coil),  
 $X_{\text{Pet}}$  - reactance of the arc suppression coil (Petersen coil),  
 $m$  - degree of over-compensation of the network (in p.u.)

This value calculated according to (8) can be obtained – as in the previous section – by means of a slightly redrawn representation (similar to the system of symmetrical components) produced in Fig.16.

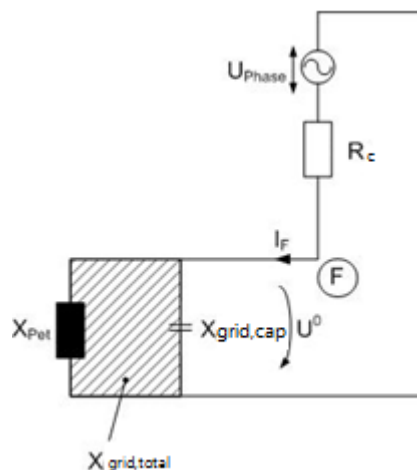


Fig. 16. Ground fault in a network with earth fault compensation; equivalent circuit

The total reactance of the network given in Fig.16 can be calculated using (9).

$$X_{\text{grid, total}} = U_{\text{Phase}} / I_F = U_{\text{Phase}} / (m \cdot I_{\text{grid, cap, e/f}}) \quad (9)$$

**Remark:** The value for  $X_{\text{grid,total}}$  can also be obtained for the control purposes from the following relationship

$$X_{\text{grid,total}} = X_{\text{grid,cap}} // X_{\text{Pet}} = (-U_{\text{Phase}}/I_{\text{grid,cap,e/f}}) // (+U_{\text{Phase}} / I_{\text{Pet}}) \quad (10)$$

The earth fault current must be calculated from the complete current loop

- substation busbar  $\diamond$  line to ground transition resistance  $R_c$   $\diamond$  source reactance of the network  $X_{\text{grid,total}}$   $\diamond$  substation busbar

$$I_F = (U_N/\sqrt{3}) / \sqrt{(R_c^2 + X_{\text{grid,total}}^2)} \quad (11)$$

### Numerical example 1:

Earth fault in an extended 20 kV cable network (9 feeders, 10 km length each)

Arc suppression coil (Petersen coil): 3,7% overcompensated

Line to ground transition resistance = 100  $\Omega$

$$U_{\text{Phase}} = 20.000\text{V} / \sqrt{3} = 11.547 \text{ V}$$

$$I_{\text{grid,cap,e/f}} = I_{\text{cable}} \cdot i'_{\text{cable}} + I_{\text{OHL}} \cdot i'_{\text{OHL}} = 9 \text{ feeders} \cdot 10 \text{ km} \cdot 3 \text{ A/km} + 0 = 270 \text{ A}$$

$$I_{\text{Pet}} = (1+m) \cdot I_{\text{grid,cap,e/f}} = (1 + 0,037) \cdot 270 \text{ A} = 280 \text{ A}$$

$$X_{\text{grid,total}} = U_{\text{Phase}} / I_F = U_{\text{Phase}} / (m \cdot I_{\text{grid,cap,e/f}}) = 11.547 \text{ V} / 10 \text{ A} = 1155 \Omega$$

$$I_F = U_{\text{Phase}} / \sqrt{(R_c^2 + X_{\text{grid,total}}^2)} = 11.547 / \sqrt{(100^2 + 1155^2)} = 9,9 \text{ A}$$

## 1.8 Calculation of ground-fault current in a network with resonant grounding – calculation of the 250-Hz-components

When calculating the 250-Hz conditions the following has to be taken into account (see Fig.17):

1. The driving voltage in this case is the 250-Hz component of pre-fault phase-to-earth voltage and not the 50-Hz phase voltage  $U_{\text{Phase,250}} = p_{250} \cdot U_N/\sqrt{3}$ , where  $p_{250}$  is the level of the 5th harmonic in the line voltage.
2. All capacities have at 250 Hz a reactance that is 5 times smaller than their reactance at 50 Hz.
3. The Petersen coil has at 250 Hz a reactance that is 5 times bigger than their reactance at 50 Hz. Therefore, its reactance in relation to the system capacitances can be neglected due to the parallel circuit
4. The reactances of the lines are 5 times as large as at 50 Hz and should not be neglected.

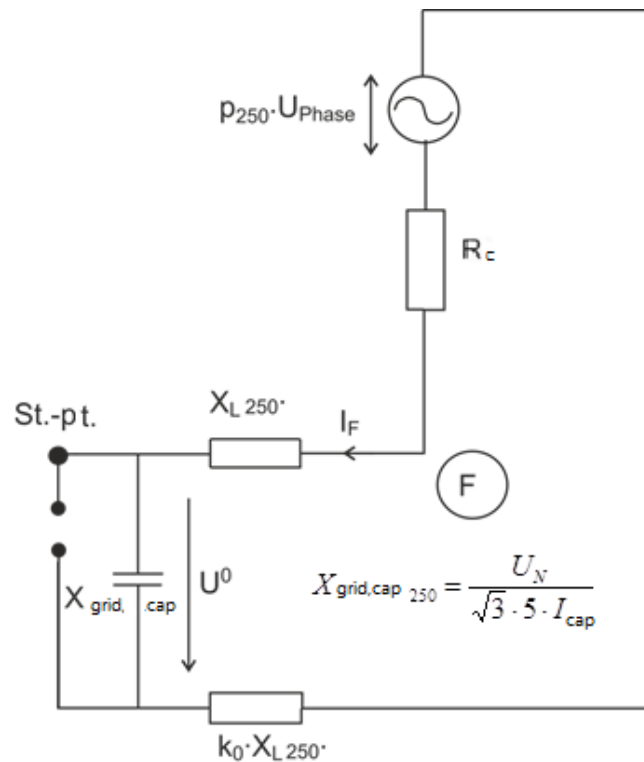


Fig. 17. Ground fault in a network with earth fault compensation; equivalent circuit

In accordance with Fig.17 the parallel circuit of the Petersen coils  $X_{pet}$  and the internal network reactance  $X_{grid,cap}$  is omitted because of the negligibility of the now high impedance of the Petersen coil. This means that at higher frequencies a network with earth fault compensation behaves like a network with isolated neutral. The line reactances are to be taken into account by the factor  $f / 50\text{Hz}$ .

**Remark 1:** Due to the simplification (from the whole line impedance only the line reactance is taken) at a certain line length the reactive components mutually cancel which leads to a resonance condition. In such a case the idealized current would become infinite, but this is unrealistic. In this case, the effective resistances are still present, which attenuate the harmonic content in the earth fault current effectively. Only in this case, an exact calculation is required.

**Remark 2:** The value for  $X_{grid,total}$  can be obtained for control purposes from the following relationship:

$$X_{grid,total,250} = X_{grid,cap50} / 5 \parallel X_{pet} \cdot 5 \approx (-U_{Phase} / I_{grid,cap,e/f}) / 5 \quad (12)$$

**Remark 3:** The reactance value of the loop from the fault location to the substation busbar and back has for 250 Hz conditions a value that is five times as high as the 50-Hz reactance. The earth fault current is to be calculated from the complete current loop:

- substation busbar  $\diamond$  line to ground transition resistance  $R_c$   $\diamond$  phase conductor  $X_{L,250}$   $\diamond$  network capacity  $X_{grid,total,250}$   $\diamond$  earth return path impedance  $Z_{m,250} = k_0 \cdot X_{L,250}$   $\diamond$  substation busbar

$$I_{F,250} = p_{250} \cdot (U_N / \sqrt{3}) / \sqrt{(R_C^2 + [(1+k_0) \cdot X_{L,250} - X_{\text{grid, cap, 250}}]^2)} \quad (13)$$

### Numerical example 2:

Earth fault in an extended 20 kV cable network (9 feeders, 10 km length each with  $x' = 0,13 \Omega/\text{km}$ )

Arc suppression coil (Petersen coil): 3,7% overcompensated (does not matter in the following)

Line to ground transition resistance = 100  $\Omega$

Fault location 3 km away from the substation busbar

$$X_L = 3 \text{ km} \cdot 0,13 \Omega/\text{km} = 0,39 \text{ Ohm}$$

$$k_0 = 0,7$$

$$p_{250} = 2\%$$

$$I_{\text{grid, cap, e/f}} = I_{\text{cable}} \cdot i'_{\text{cable}} + I_{\text{OHL}} \cdot i'_{\text{OHL}} = 9 \text{ feeders} \cdot 10 \text{ km} \cdot 3 \text{ A/km} + 0 = 270 \text{ A}$$

$$U_{\text{Phase, 250}} = p_{250} \cdot U_N / \sqrt{3} = 0,02 \cdot 20.000 \text{ V} / \sqrt{3} = 231 \text{ V}$$

$$X_{\text{grid, total, 250}} \approx (-U_{\text{Phase}} / I_{\text{grid, total}}) / 5 = (11547 \text{ V} / 270 \text{ A}) / 5 = 8,6 \Omega$$

$$X_{L, 250} = x' \cdot l \cdot 5 = 3 \text{ km} \cdot 0,13 \Omega \cdot 5 = 1,95 \Omega$$

$$\begin{aligned} I_{F, 250} &= p_{250} \cdot (U_N / \sqrt{3}) / \sqrt{(R_C^2 + [(1+k_0) \cdot X_{L, 250} - X_{\text{grid, cap, 250}}]^2)} \\ &= 231 / \sqrt{(100^2 + [(1+0,7) \cdot 1,95 - 8,6]^2)} = 2,3 \text{ A} \end{aligned}$$

## 1.9 Calculation of the ground-fault current in a network with resonant grounding and short term resistive grounding

In resonant grounded networks in some cases, a more or less low-ohmic resistance is briefly switched parallel to the Petersen coil to improve the earth fault location by means of the evaluation of the resistive component. In this case, taking Fig.18 as a starting point, a resistor parallel to the Petersen has to be taken into account. Hence another current path is activated through this neutral point resistor causing the resistive component  $I_R$ . In Fig.19 the first physical current flow and in Fig.20 the derived circuit diagram is shown.

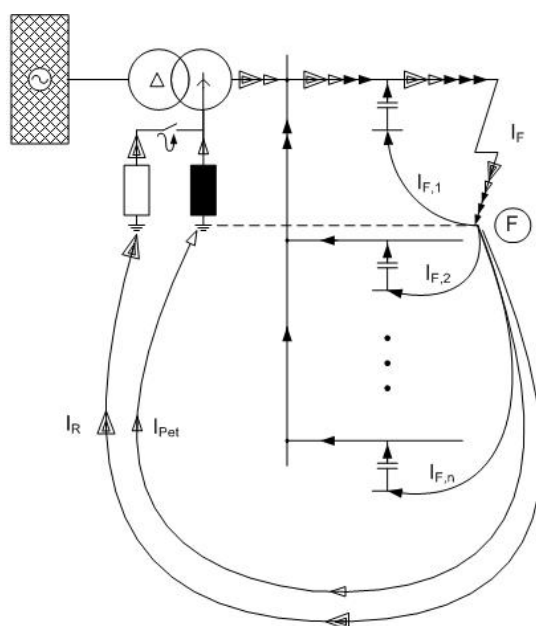


Fig. 18. Physical earth fault current flow in a network with resonant grounding and short-term resistance grounding of the neutral point; F ... fault location

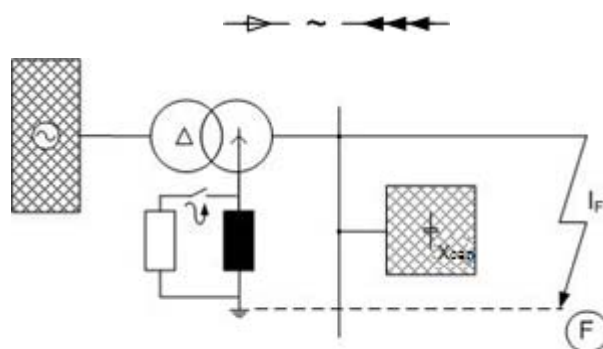


Fig. 19. Earth fault in a network with resonant grounding and short-term resistance grounding of the neutral point -Electrical diagram; F ... fault location

According to Fig.20, the ground fault current  $I_F$  is composed of the capacitive current contributions of the individual sections of the network ( $I_{grid, cap, e/f}$ ), the inductive coil current  $I_{Pet}$  and the current through the neutral point resistance  $I_R$ .

When it comes to real network technology, the resistive current  $I_R$  becomes so large that the line impedance to the fault point "F" can – in contrast to the previous sections - no longer be neglected.

In analogy to the previous section the method of symmetrical components is taken as starting point for the simplified representation.

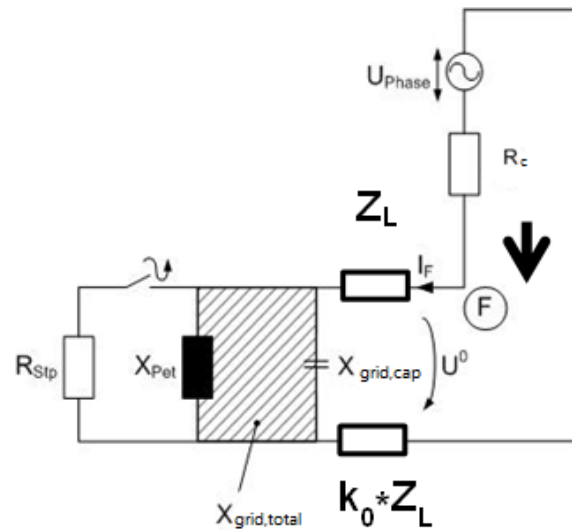


Fig. 20. Earth fault in a network with resonant grounding and short-term resistance grounding of the neutral point –equivalent circuit diagram; F ... fault location

To answer the core question of the earth fault calculation "How is the current path of the earth fault current closed, while looking at the return part of the current loop from ground into the elsewhere highly galvanically isolated power network?" the following current loop has to be considered (see Fig.20).

- substation busbar  $\diamond$  line to ground transition resistance  $\diamond$  phase conductor  $Z_L$   $\diamond$  parallel connection of the source impedance of the network  $X_{\text{grid,total}}$  (network capacity // Petersen coil inductance) and the neutral point resistor  $R_{\text{Stp}}$   $\diamond$  line to ground transition resistance  $R_c$   $\diamond$  earth return path impedance  $Z_m = k_0 Z_L$   $\diamond$  star point resistor  $R_{\text{Stp}}$   $\diamond$  substation busbar

$$I_F = U_{\text{Phase}} / (Z_L + R_c + Z_m + X_{\text{grid,total}} // R_{\text{Stp}}) \quad (14)$$

$$I_F = U_{\text{Phase}} / [(R_{\text{Stp}} // X_{\text{grid,total}}) + Z_L(1+k_0) + R_c] \quad (15)$$

where:

$R_{\text{Stp}}$	- star point resistor,
$X_{\text{grid,total}}$	- source impedance of the network
	$X_{\text{grid,total}} = U_{\text{Phase}} / (m \cdot I_{\text{grid,cap,e/f}}),$
$m$	- degree of overcompensation of the network (p.u.),
$Z_L$	- Impedance of the phase conductor,
$Z_m$	- Earth return path impedance of "ground",
$R_c$	- line to ground transition resistance at the fault location,
$k_0$	- Ratio between the earth return path impedance and the impedance of the phase conductor

## 2. CONCLUSION

By means of the simplified procedure for the calculation of the earth fault current, statements for medium voltage networks can be made lightly with tolerable imprecision. The results of the method shown above permit on one hand a facilitated access to this task and permit simplification of the necessary calculations of ground fault currents.

## 3. REFERENCES

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